Korteste vei alle-til-alle

TDT4120
Algoritmer og datastrukturer

2011-14-10

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Dagens plan

- Neste ukes øving
- Korteste vei alle-til-alle
  - Floyd-Warshall
- Denne ukens øving

Neste ukes øving

- Praksis – Kortstokker
  - Fletting av ferdigsorterte lister
- Teori – Korteste vei

Korteste vei alle-til-alle

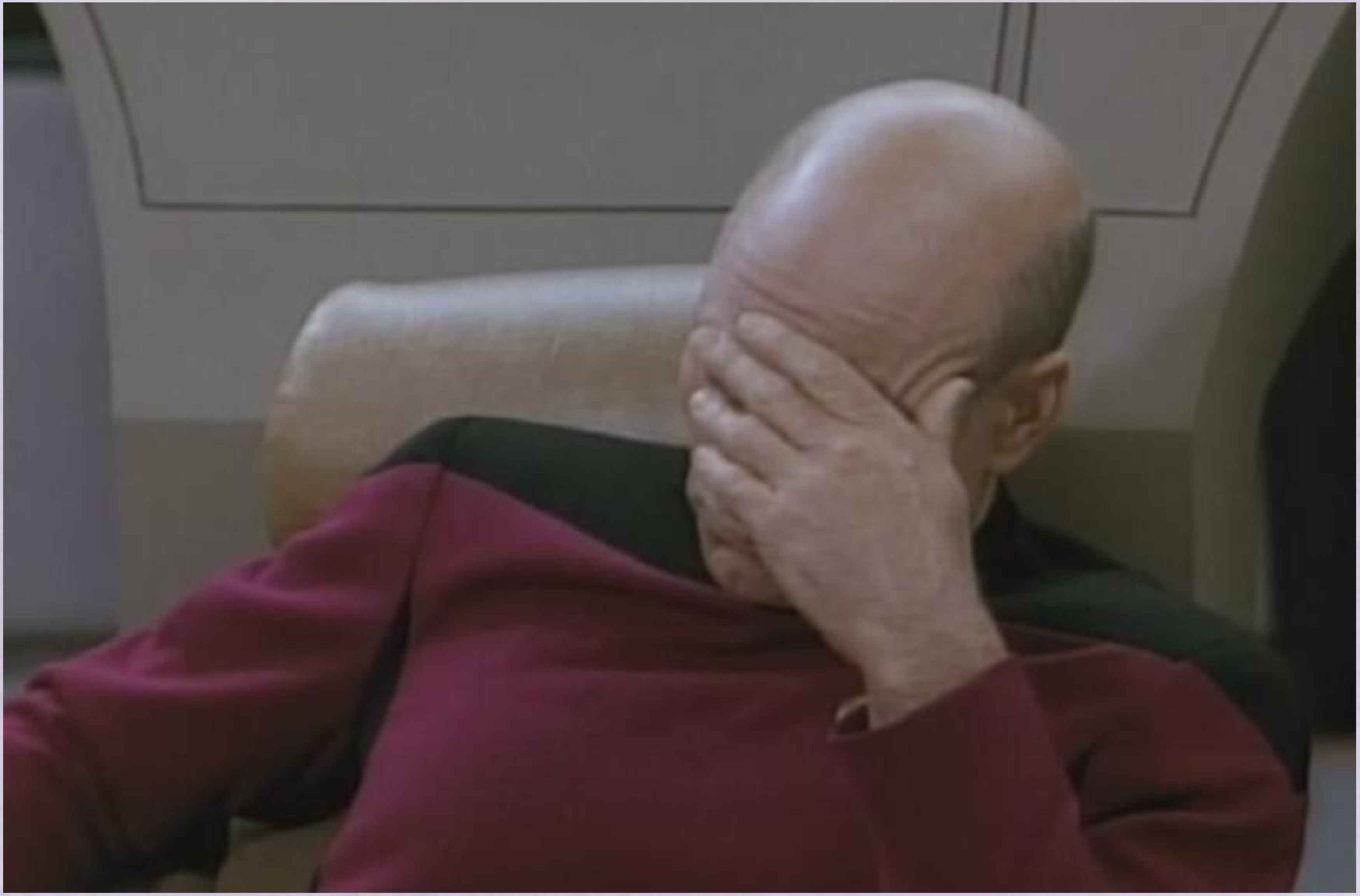
Korteste vei – mange muligheter

Én-til-én	Alle-til-én
Én-til-alle	Alle-til-alle

Korteste vei alle-til-alle – problem solved

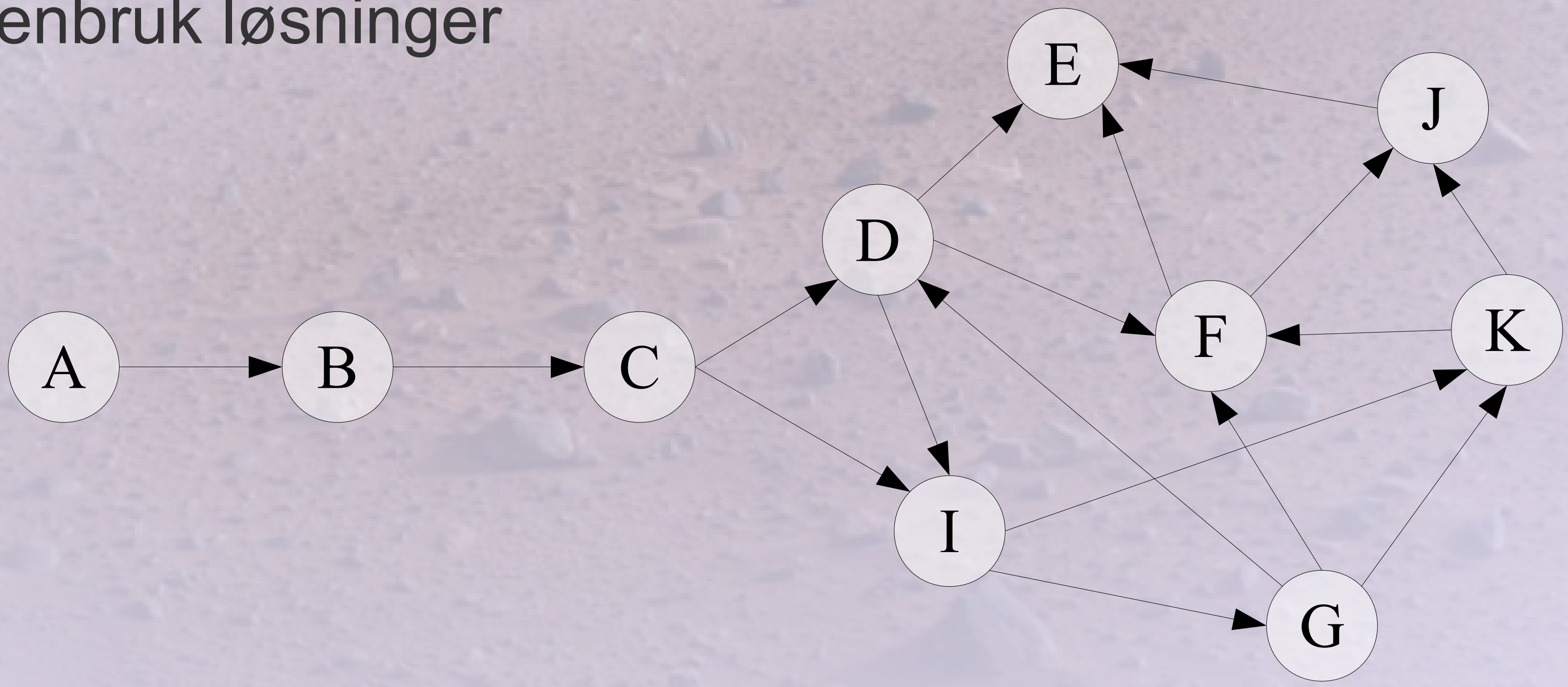
- Kjør en én-til-alle korteste vei algoritme fra alle nodene
  - F.eks Djikstra eller Bellman-Ford
- Kjøretid Dijkstra
  - Én-til-alle: $\Theta(E \lg V)$
  - Alle-til-alle: $\Theta(VE \lg V)$
- Ferdig for i dag :)

Korteste vei alle-til-alle



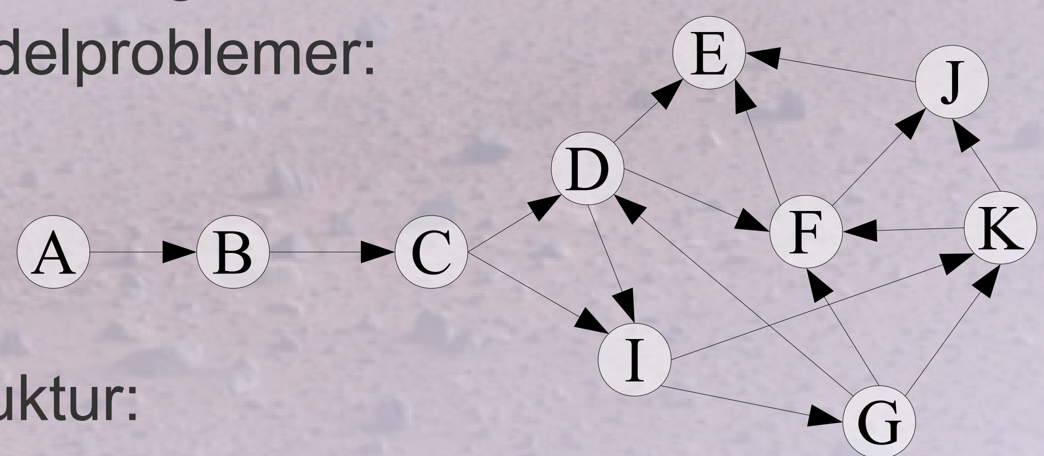
Korteste vei alle-til-alle

- Kan gjøres bedre
- Kjører vi én-til-alle fra alle nodene løser vi sannsynligvis det samme problemet flere ganger
- Gjenbruk løsninger



Korteste vei alle-til-alle

- Korteste-vei-problemet har **overlappende delproblemer** og **optimal substruktur**
- Viktig kjennetegn på problemer som kan løses vha. **dynamisk programmering**
- I en korteste-vei-setting:
  - Overlappende delproblemer:



  - Optimal substruktur:

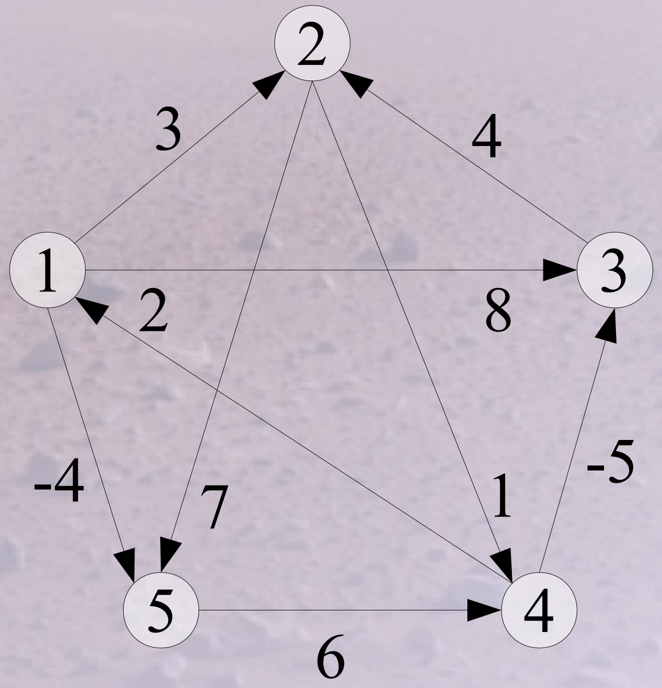
Dagens antagelser

- Nabomatriser
- Ingen negative sykler
  - men negative kanter er lov
- $n = |V|$
- $\delta(i, j) =$ Korteste veien fra i til j

Naboliste vs nabomatrise

$$\begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

1	(2,3), (3,8), (4,-4)
2	(4,1), (5,7)
3	(2,4)
4	(1,2), (3,-5)
5	(4,6)

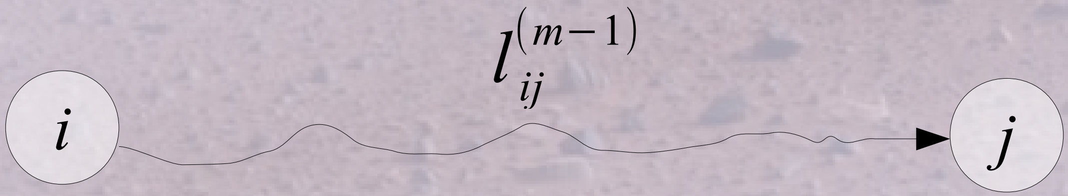


Korteste vei alle-til-alle

Andre forsøk

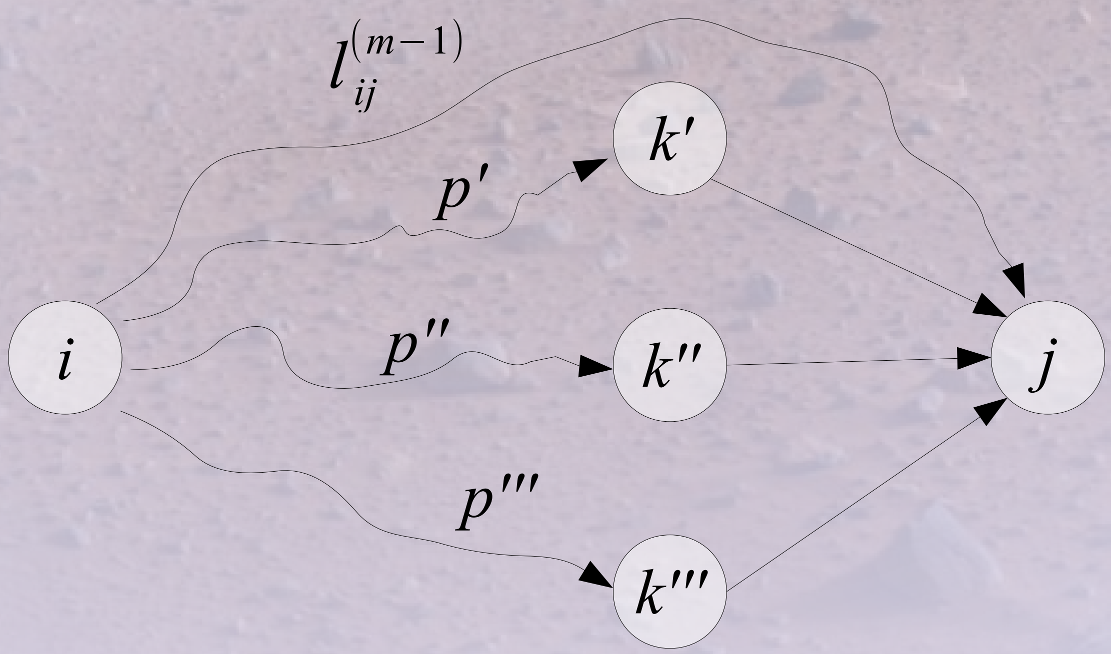
Korteste vei alle-til-alle

- Ønsker å finne korteste vei mellom node i og node j
  - Anta noen har jobbet på problemet før
  - Har funnet en mulig korteste vei
  - Men har kun fått sett på stier bestående av maks $m-1$ kanter



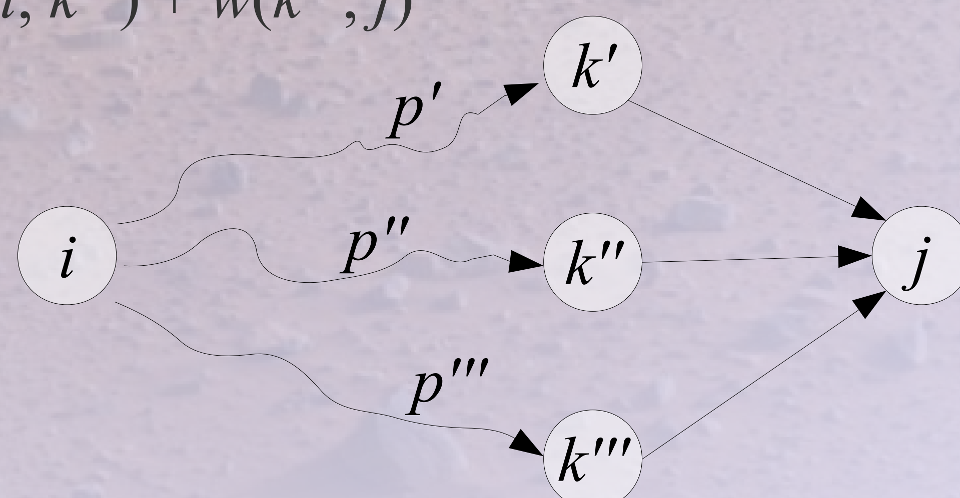
- Hva gjør du?

- Ser på stier bestående av kanter m kanter
- Hvilke alternativer eksisterer?
  1. Forgjengeren fant det beste valget, dvs: $l_{ij}^{(m)} = l_{ij}^{(m-1)}$
  2. Bedre å bruke m kanter enn $m-1$
     - Må finne en sti med $m-1$ kanter hvor endenoden har en kant til j



Korteste vei alle-til-alle

- Skaffet oss tre nye problemer:
  - Korteste vei fra i til k'
  - Korteste vei fra i til k''
  - Korteste vei fra i til k'''
- Må sjekke alle mulighetene:
  - $w(p') + w(k', j) = w(i, k') + w(k', j)$
  - $w(p'') + w(k'', j) = w(i, k'') + w(k'', j)$
  - $w(p''') + w(k''', j) = w(i, k''') + w(k''', j)$
  - Hvilken er billigst?



Korteste vei alle-til-alle

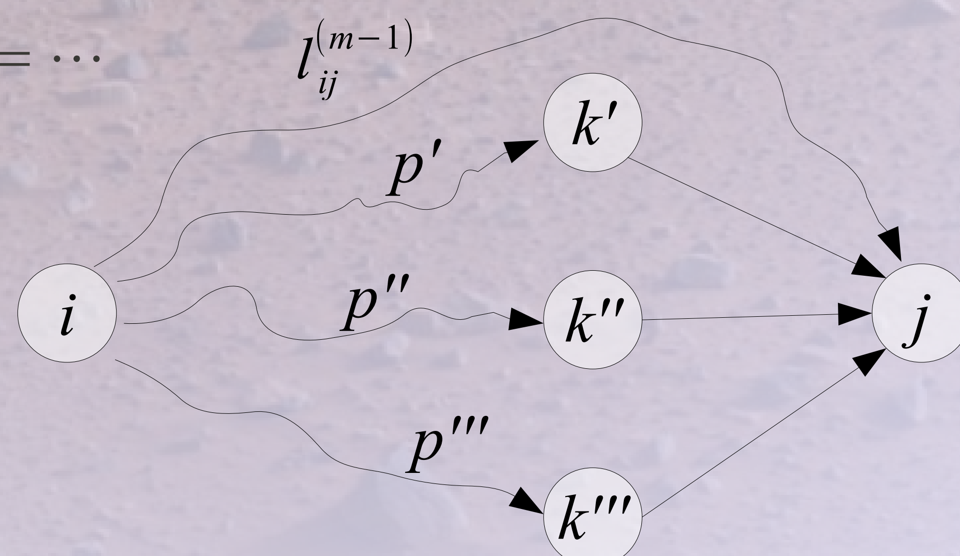
- Alle disse stiene består av $m - 1$ kanter
  - Forgjengeren vår har allerede det problemet!

$$l_{ij}^{(m)} = \min_{1 \le k \le n} \{ l_{ik}^{(m-1)} + w_{kj} \}$$

- Tar vi med alternativ 1:

$$l_{ij}^{(m)} = \min \left(l_{ij}^{(m-1)}, \min_{1 \le k \le n} \{ l_{ik}^{(m-1)} + w_{kj} \} \right)$$

$$\delta(i, j) = l_{ij}^{(n-1)} = l_{ij}^{(n)} = l_{ij}^{(n+1)} = \cdots$$



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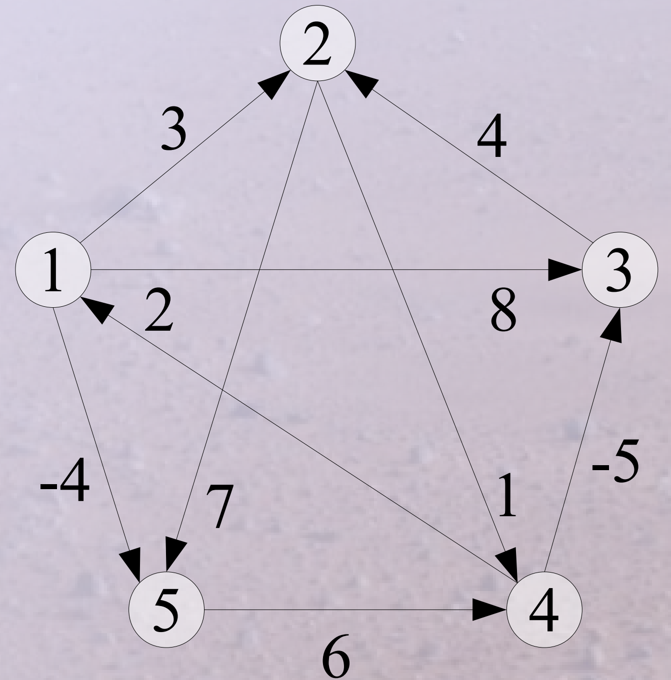


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let L' = ( l'_ij ) be a new n x n matrix
for i = 1 to n
    for j = 1 to n
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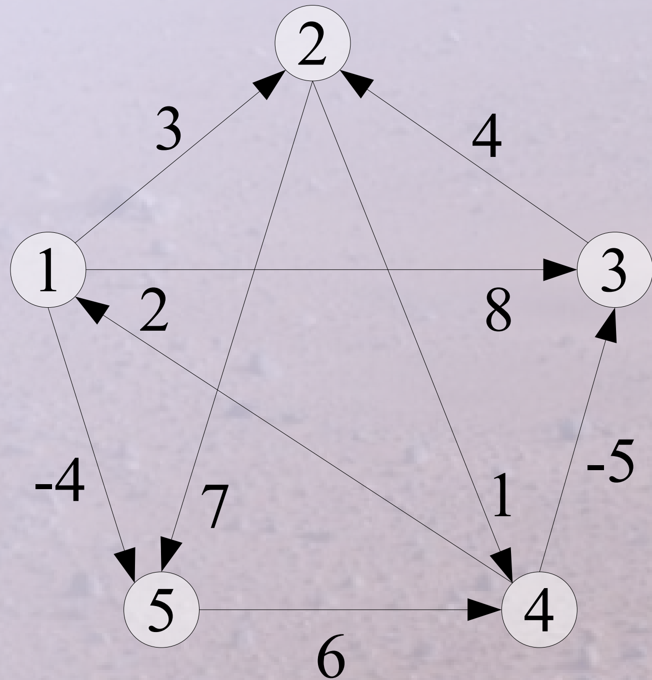
$$L^{(1)} = \begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

$$L' = \begin{pmatrix} \\ \\ \\ \\ \\ \end{pmatrix}$$

Korteste vei alle-til-alle

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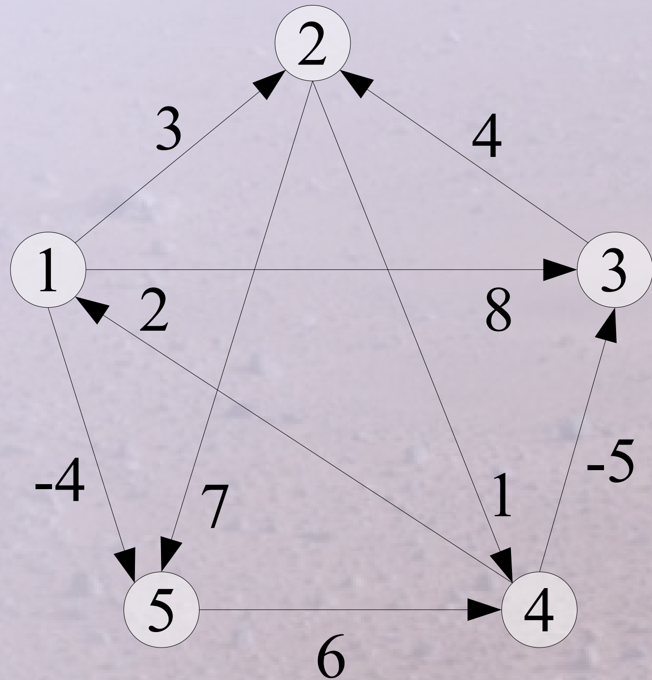


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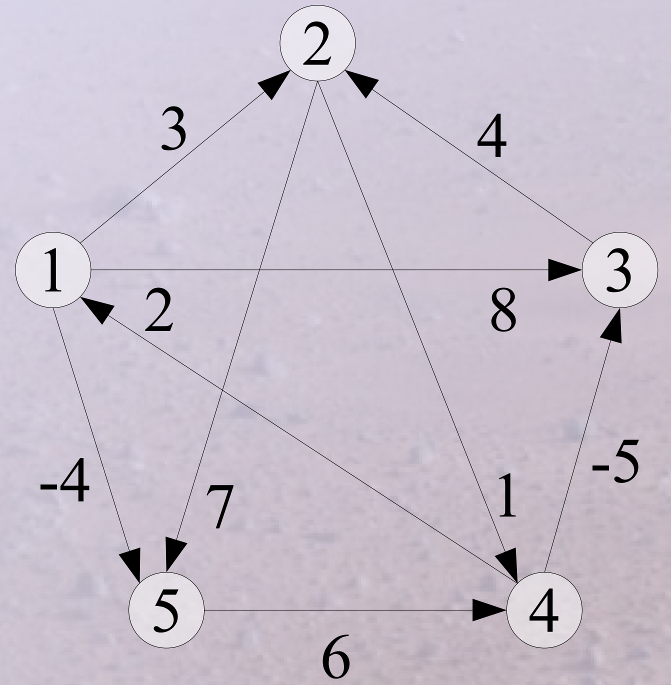


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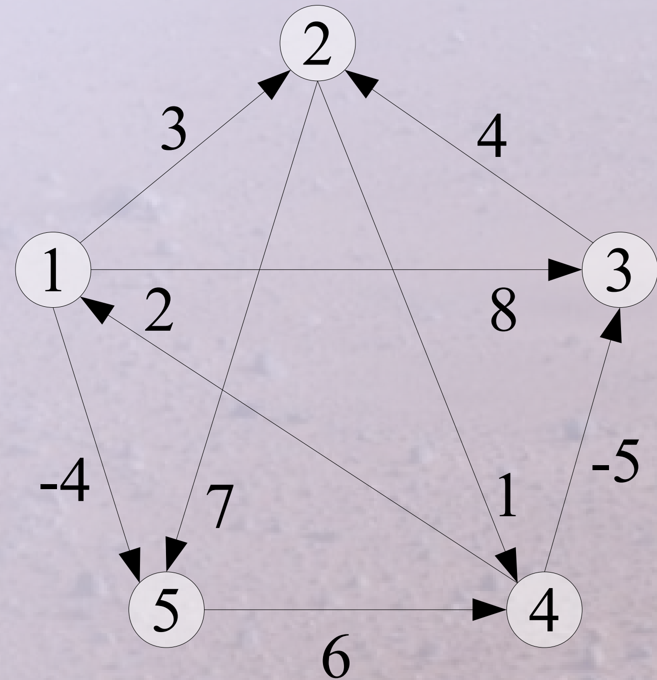


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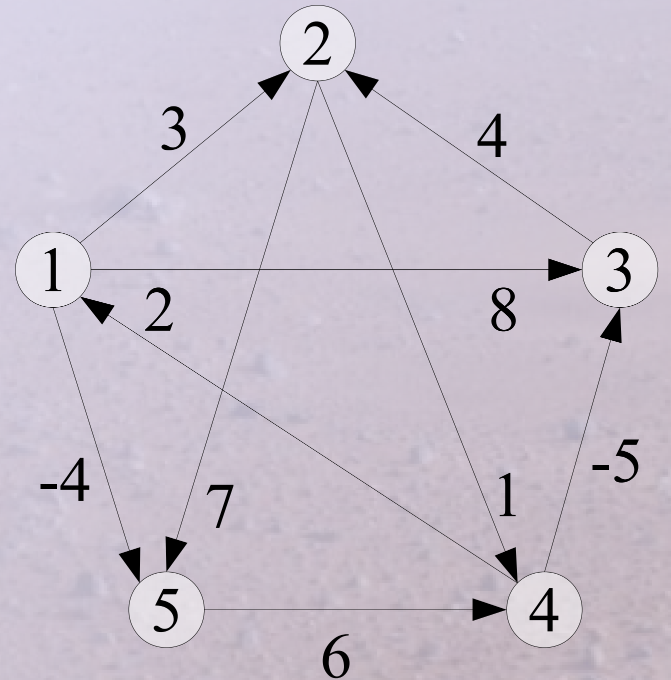


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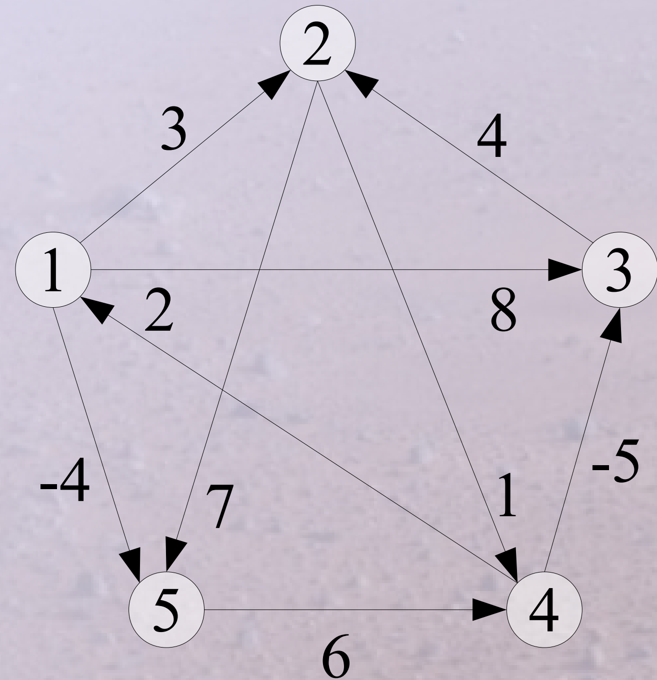


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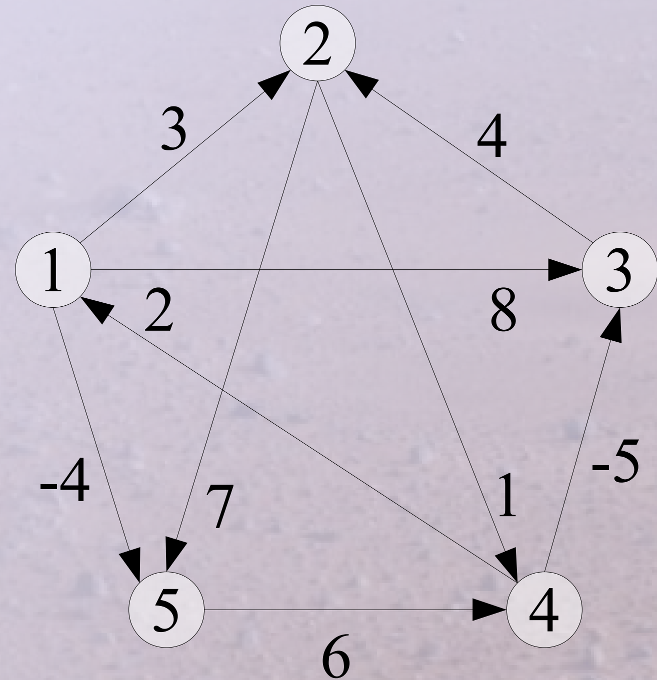

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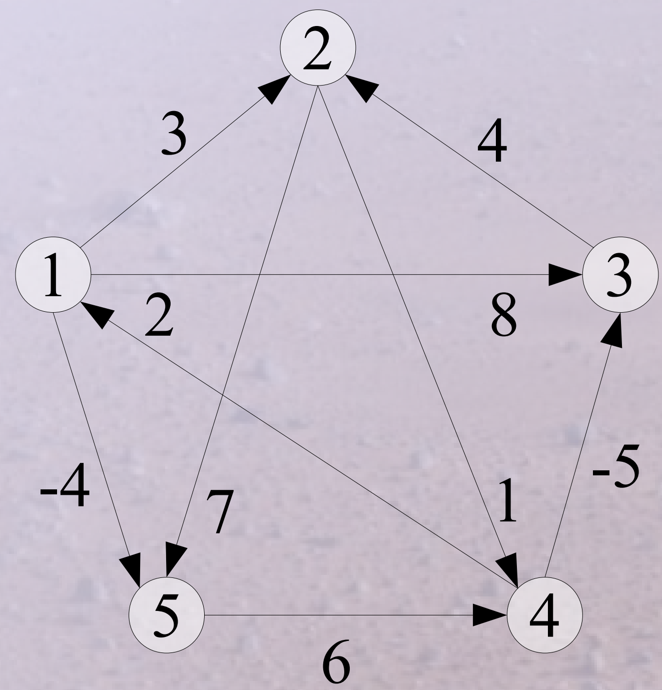


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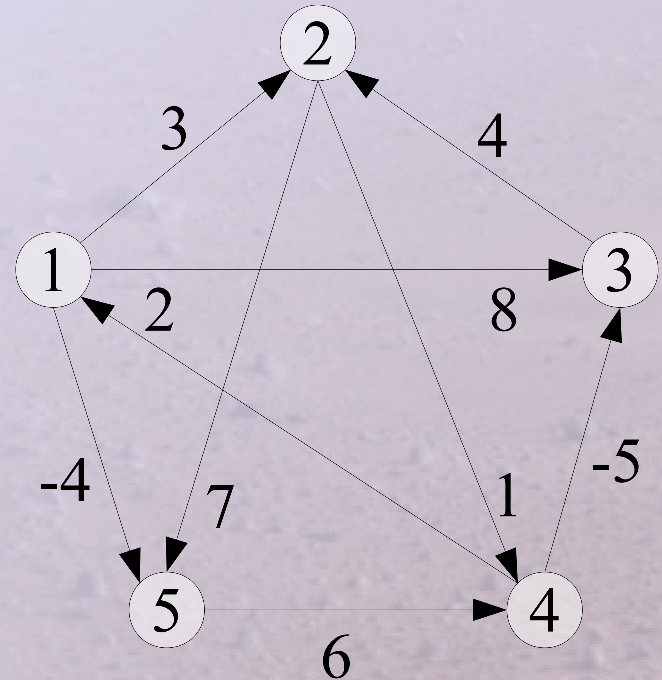


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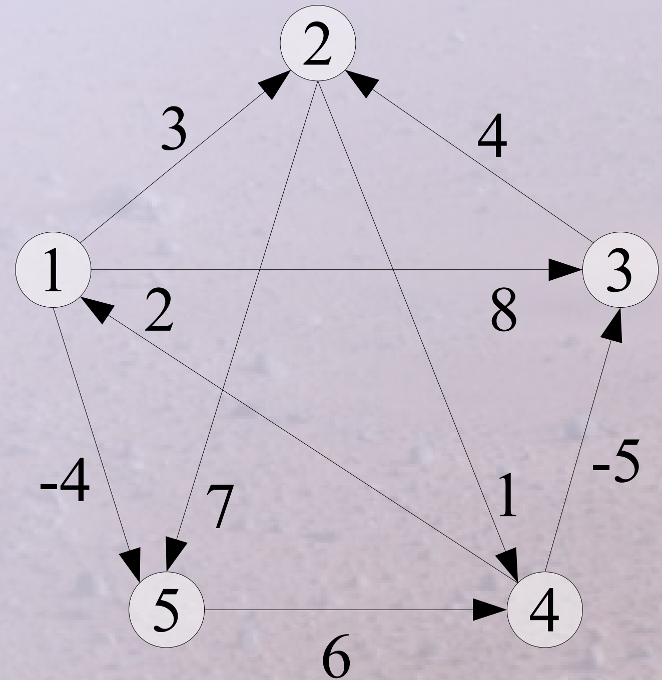


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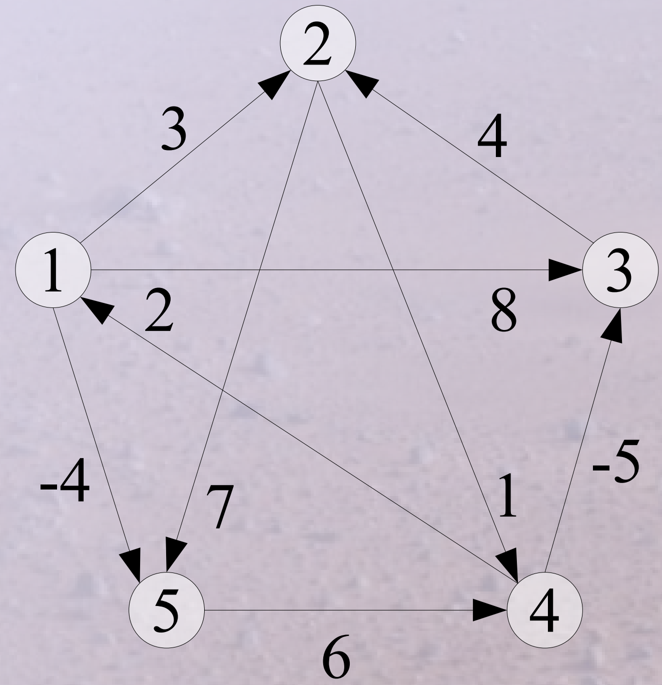


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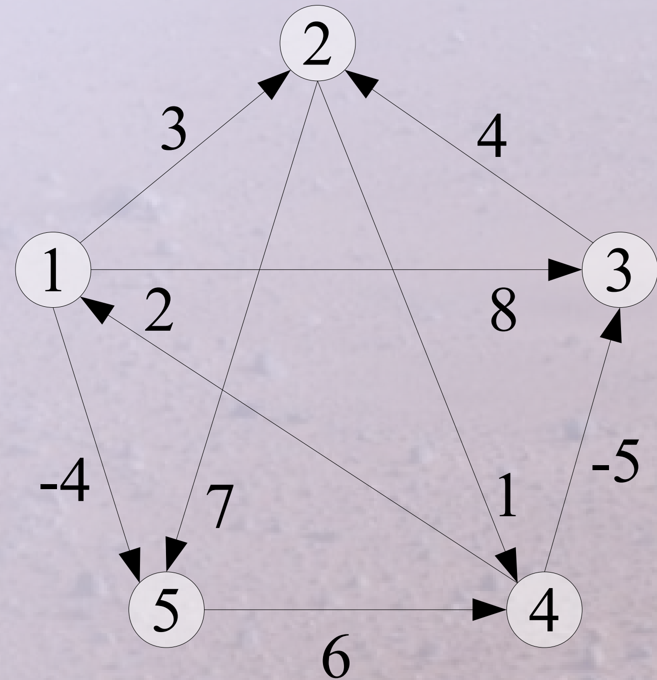


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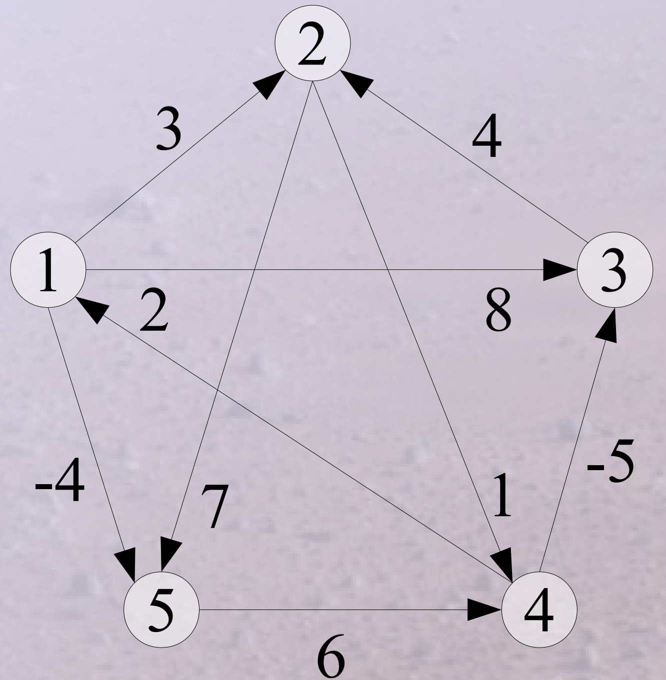


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$$L' = \begin{pmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & 2 \\ 8 & \infty & 1 & 6 & 0 \end{pmatrix}$$

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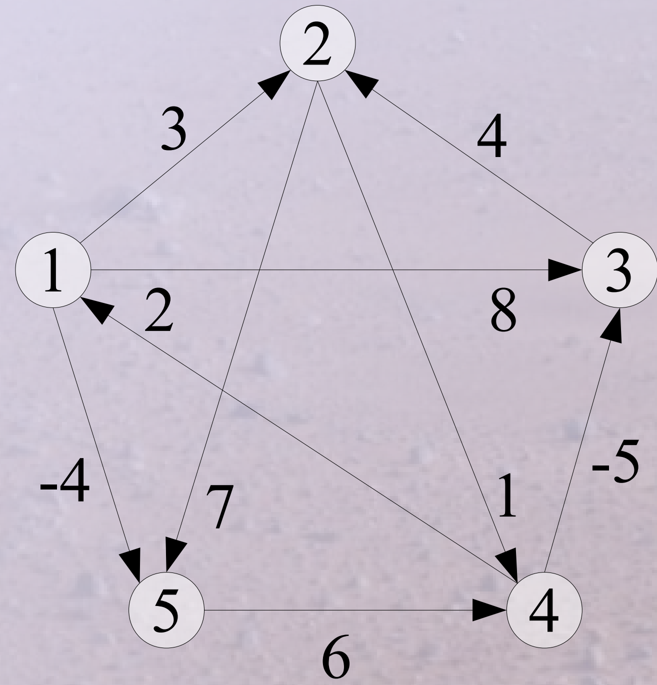


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$$L^{(2)} = \begin{pmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & 2 \\ 8 & \infty & 1 & 6 & 0 \end{pmatrix}$$

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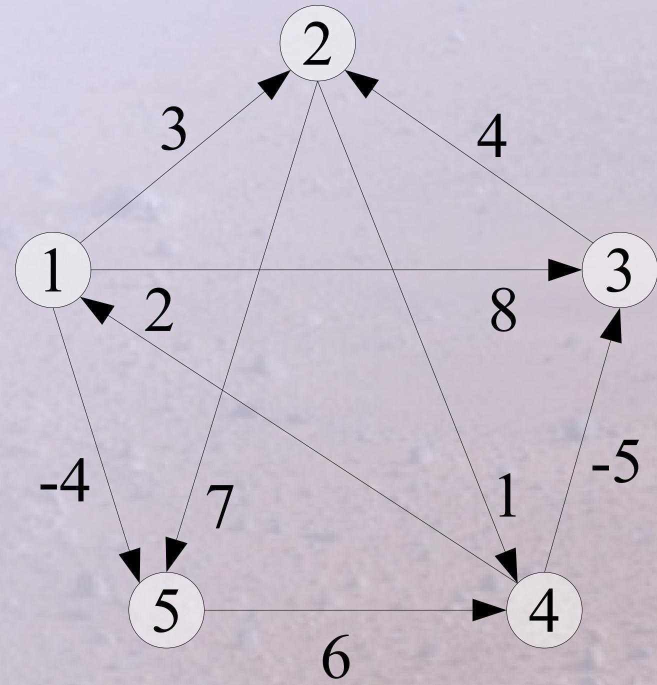

Extend-Shortest-Paths (L, W)

$n = L.rows$

let $L' = (l'_{ij})$ be a new n x n matrix

for i = 1 **to** n

  for j = 1 **to** n

    $l'_{ij} = \infty$

    for k = 1 **to** n

      $l'_{ij} = min\,(l'_{ij}, l_{ik} + w_{kj})$

return L'

$$L^{(2)} = \begin{pmatrix} 0 & 3 & 8 & 2 & -4 \\ 3 & 0 & -4 & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & 2 \\ 8 & \infty & 1 & 6 & 0 \end{pmatrix} \qquad L^{(3)} = \begin{pmatrix} 0 & 3 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & 2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

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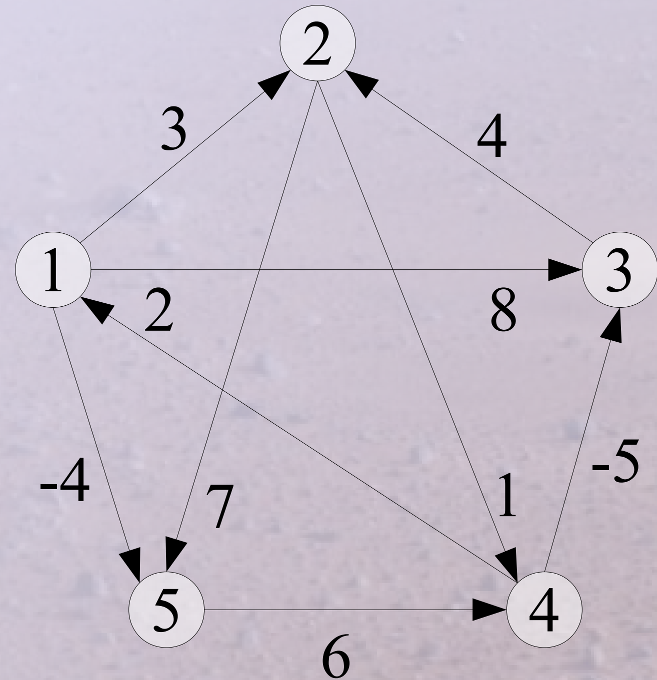


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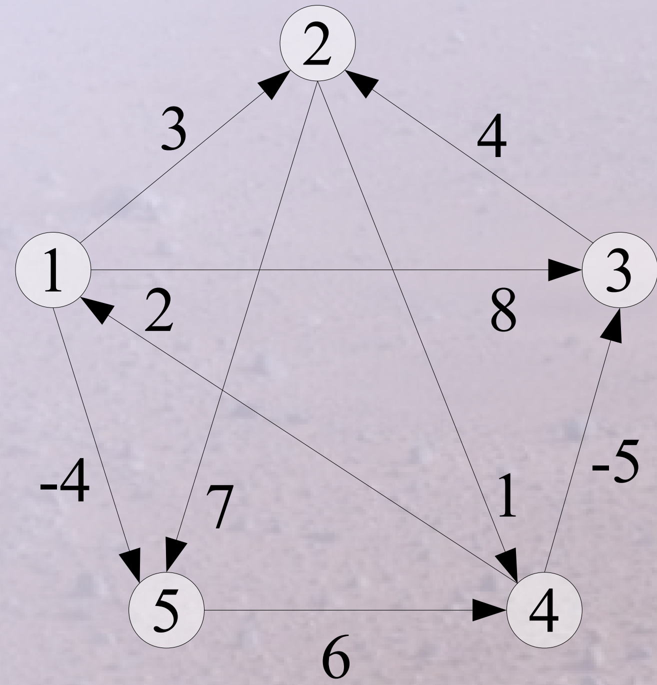


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        l′ij = ∞
        for k = 1 to n
            l′ij = min (l′ij , lik + wkj )
return L′
```

$$L^{(3)} = \begin{pmatrix} 0 & 3 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & 2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix} \qquad L^{(4)} = \begin{pmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

Korteste vei alle-til-alle

- Kjøretid: $\Theta(n^4)$
  - **Extend-Shortest-Paths** kjøres $n - 2$ ganger
- Dårligere enn Dijkstra fra hver node som er: $\Theta(VE \lg V)$
- Vi er kun interessert i $L^{(n-1)}$, men kalkulerer alle $L^{(1)}, L^{(2)}, L^{(3)}, \ldots$
  - Er det nødvendig?
  - Nei! Kan doble antall kanter vi tillater på en sti etter hver gang vi har kjørt **Extend-Shortest-Paths**
  - $L^{(1)}, L^{(2)}, L^{(4)}, L^{(8)}, \ldots L^{(2^m)}, \ldots$
  - Kjøretid: $\Theta(n^3 \lg n)$

```
Extend-Shortest-Paths (L, W )
  n = L.rows
  let L′ = ( l′ij ) be a new n x n matrix
  for i = 1 to n
      for j = 1 to n
          l′ij = ∞
          for k = 1 to n
              l′ij = min (l′ij , lik + wkj )
  return L′
```

Korteste vei alle-til-alle - Oppsummering

- Finn korteste vei alle-til-alle når du kun tillater m kanter på en sti
- For hver m, still deg spørsmålet:
  - "Kan jeg finne en billigere vei fra i til j, enn hva jeg allerede har klart med $m - 1$ kanter?"
- Avslutt når $m = n - 1$
  - Tweak: ikke alle verdier mellom 1 og $n - 1$ må vurderes

Korteste vei alle-til-alle

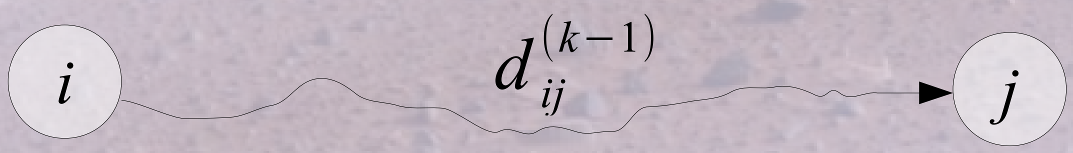
Tredje forsøk:
Floyd-Warshall

Floyd-Warshall

- En annen tilnærming til problemet
  - Forrige algoritme så på stier med maks m **kanter**
  - Floyd-Warshall ser på stier som går innom **node** k
- Også en dynamisk programmeringsalgoritme

Floyd-Warshall

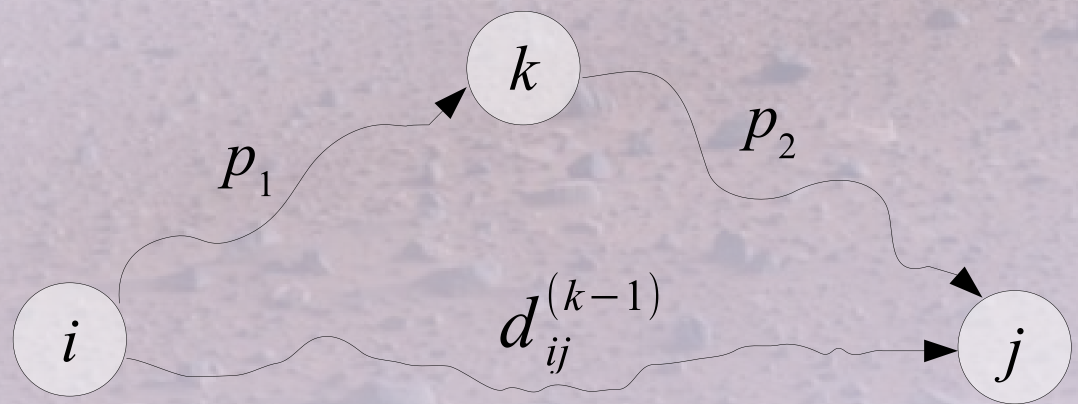
- Ønsker å finne korteste vei mellom node i og node j
  - *Igjen* har noen jobbet på problemet!
  - *Igjen* har de funnet en mulig korteste vei
  - Men har kun fått sett på stier som inneholder nodene $\{1, 2, \ldots, k-1\}$



- Hva gjør du?

Floyd-Warshall

- Ser på stier som også kan inneholde node k
- Hvilke alternativer eksisterer?
  1. Forgjengeren fant den beste veien, dvs: $d_{ij}^{(k)} = d_{ij}^{(k-1)}$
  2. Bedre å gå innom k
     - Må nå finne to korteste stier: p_1, p_2



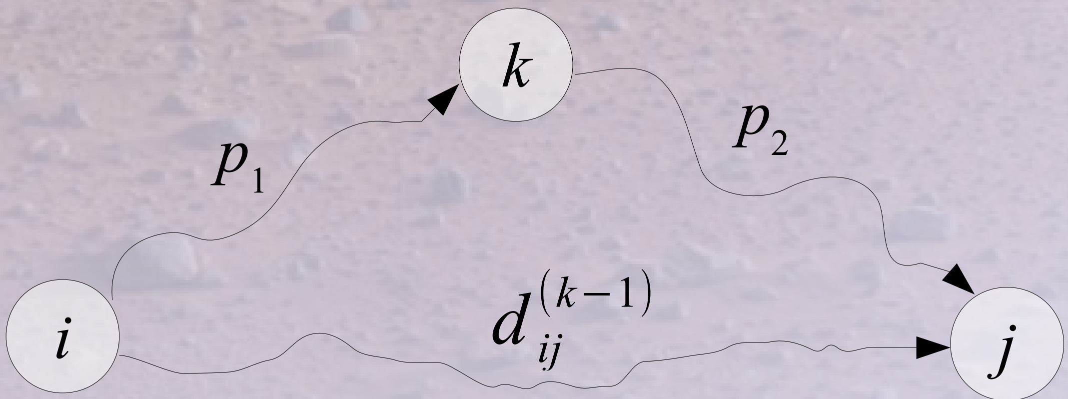
- Begge disse stiene består kun av noder i $\{1, 2, \dots, k-1\}$
  - Forgjengeren vår har allerede løst disse problemene!

$$d_{ij}^{(k)} = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$$

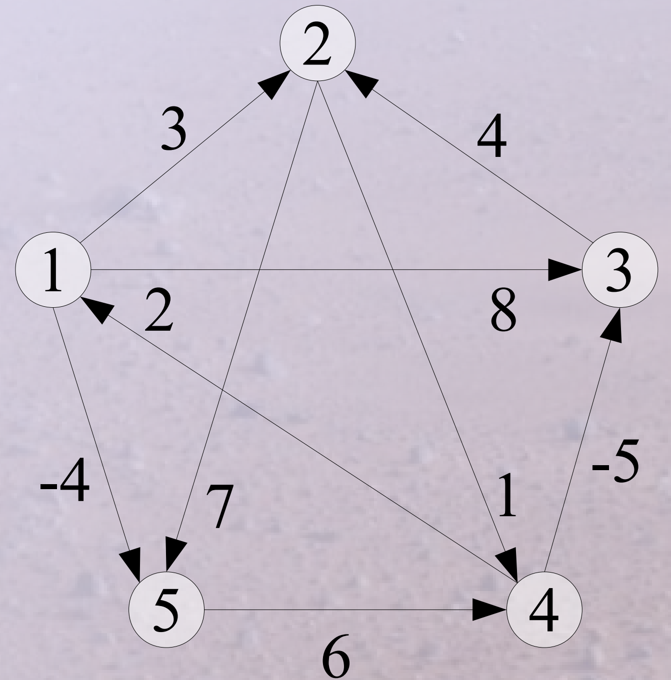
- Tar vi med alternativ 1:

$$d_{ij}^{(k)} = min\left(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$$

$$\delta(i,j) = d_{ij}^{(n)}$$



Floyd-Warshall

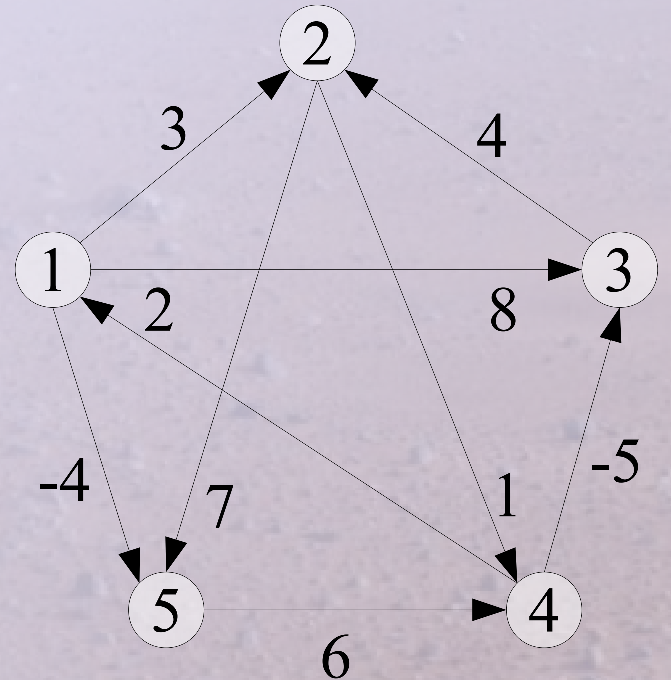



```
Floyd-Warshall (W)
n = W.rows
D = W
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            d_ij = min (d_ij , d_ik + d_kj )
return D
```

$$D=\begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

$$D^{(1)}=\begin{pmatrix} x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \\ x & x & x & x & x \end{pmatrix}$$

Floyd-Warshall

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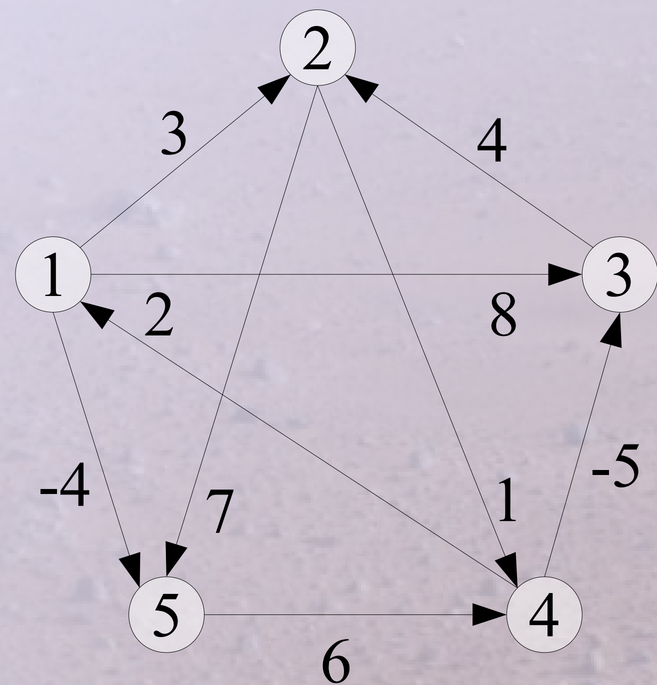
Floyd-Warshall

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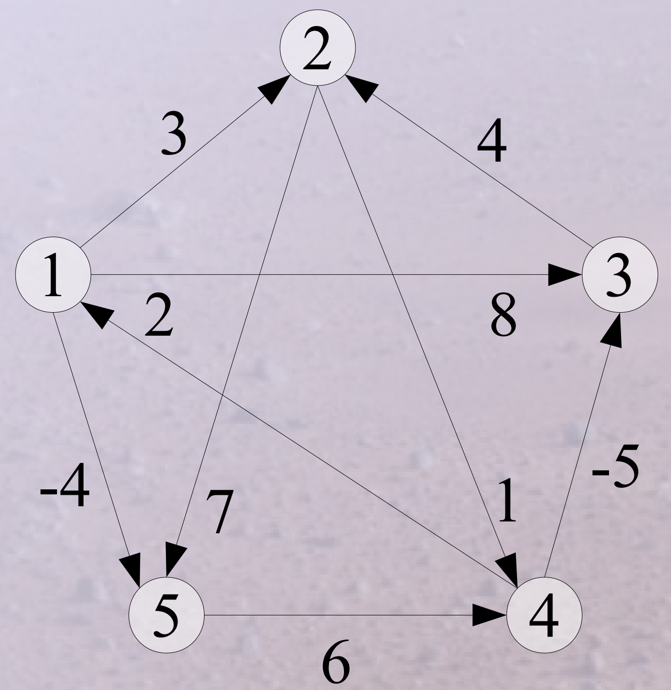
Floyd-Warshall

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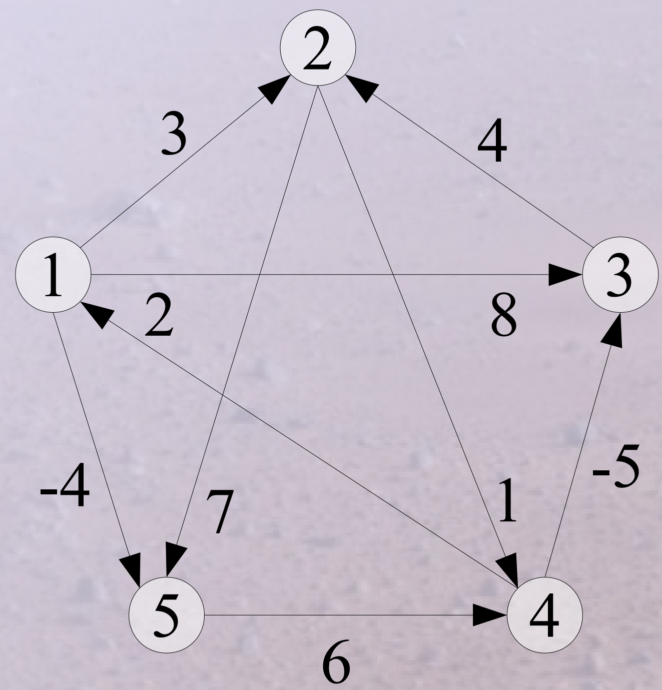
Floyd-Warshall

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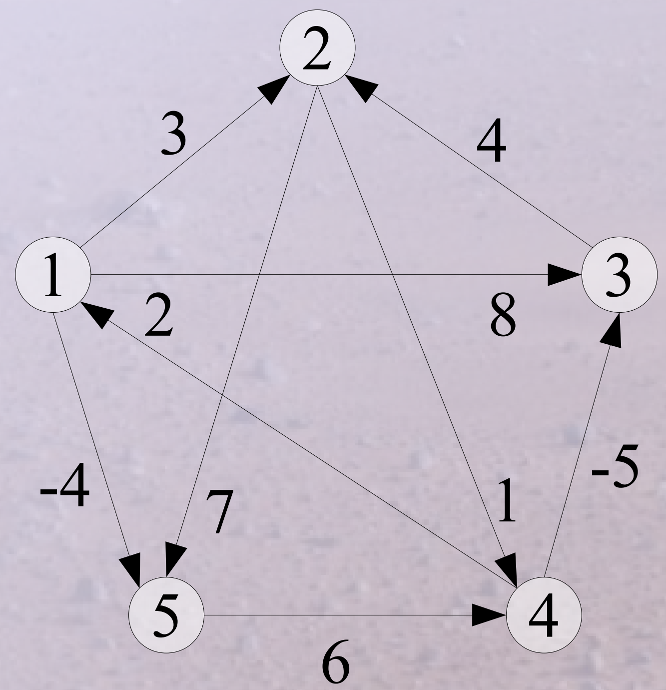
$$D^{(1)}=\begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & x & x & x & x \\ x & x & x & x & x \end{pmatrix}$$

Floyd-Warshall

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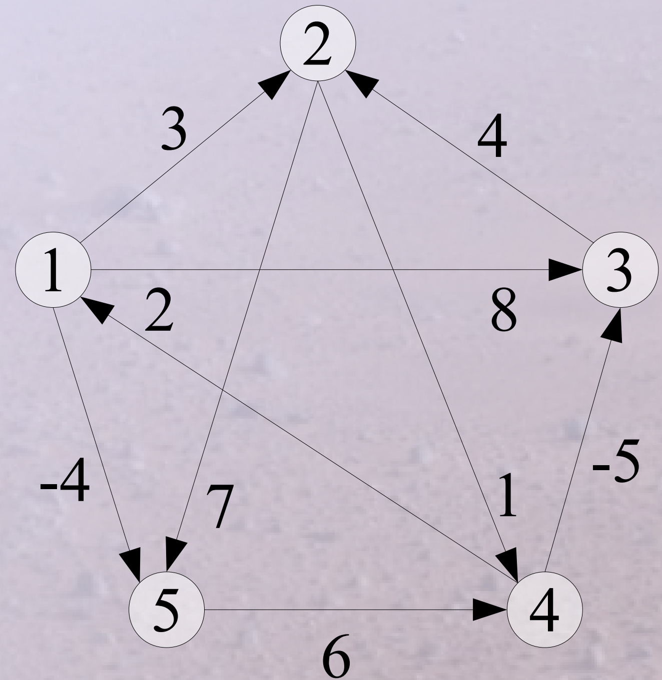
$$D^{(1)} = \begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & x & x & x \\ x & x & x & x & x \end{pmatrix}$$

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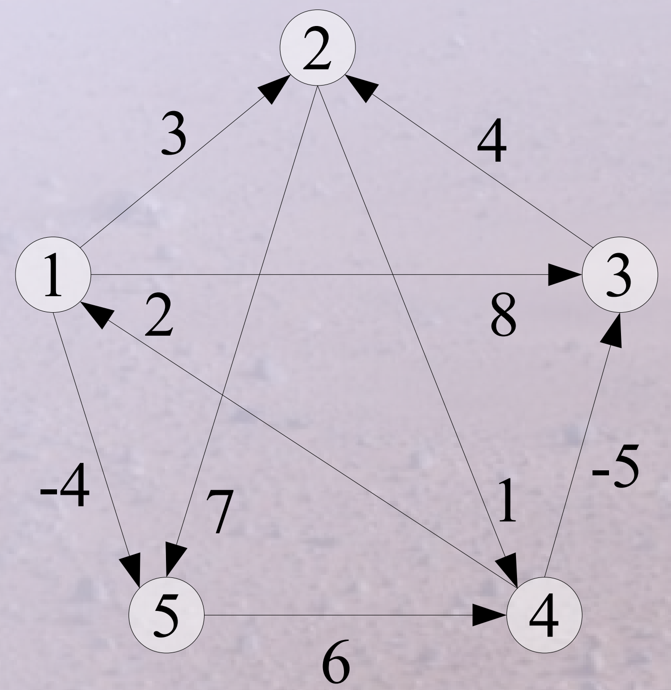
Floyd-Warshall

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```

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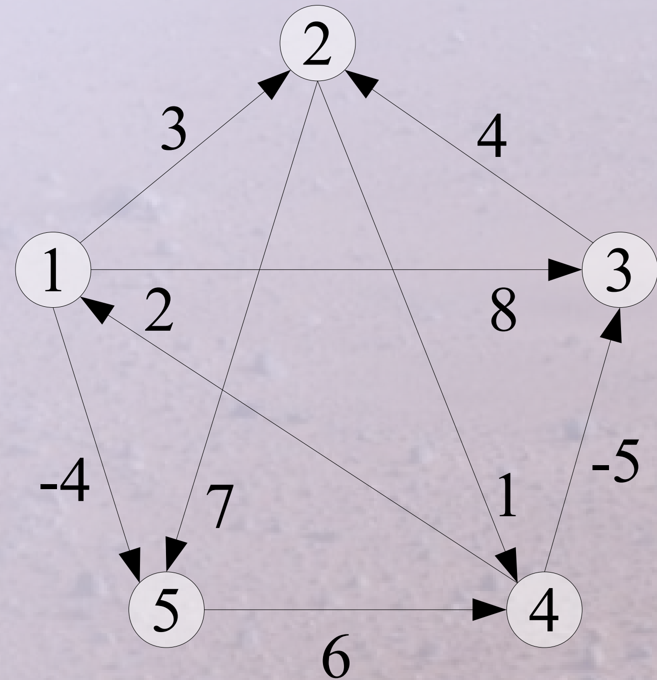
Floyd-Warshall




```
Floyd-Warshall (W)
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D = W
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    for i = 1 to n
        for j = 1 to n
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return D
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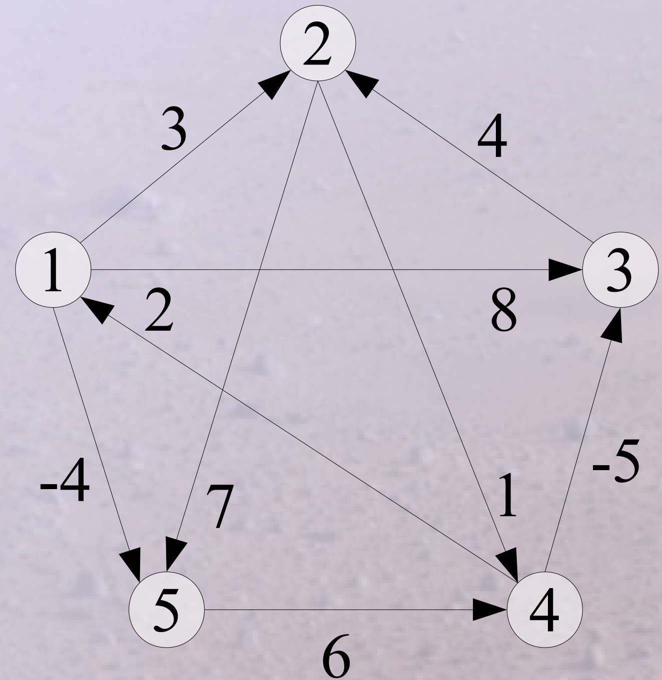
Floyd-Warshall

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  D = W
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      for i = 1 to n
          for j = 1 to n
              d_ij = min(d_ij, d_ik + d_kj)
  return D
```

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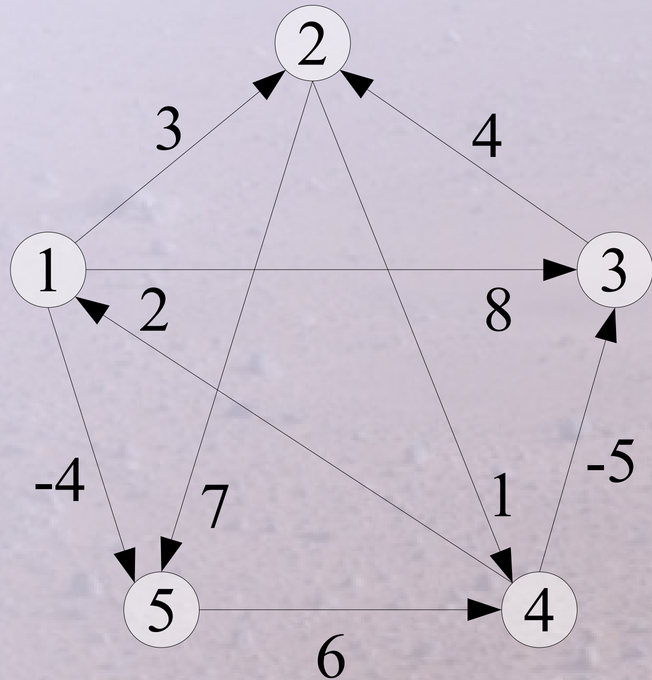


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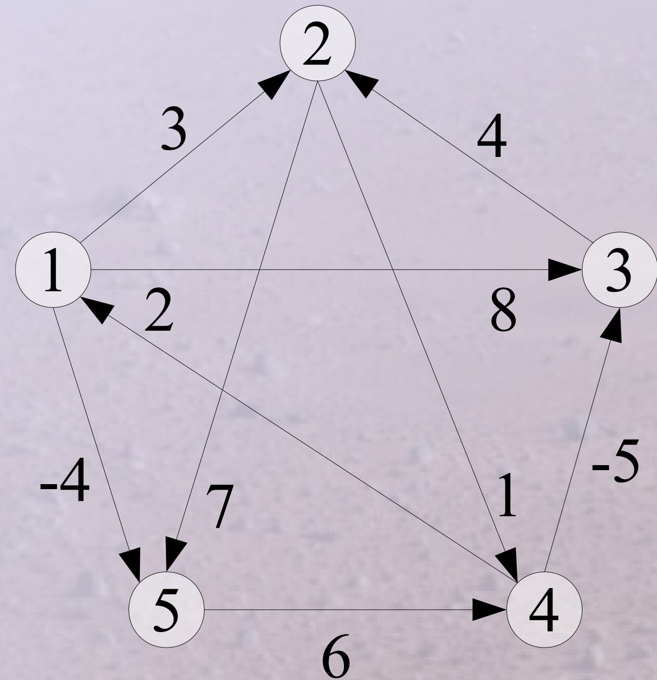
Floyd-Warshall

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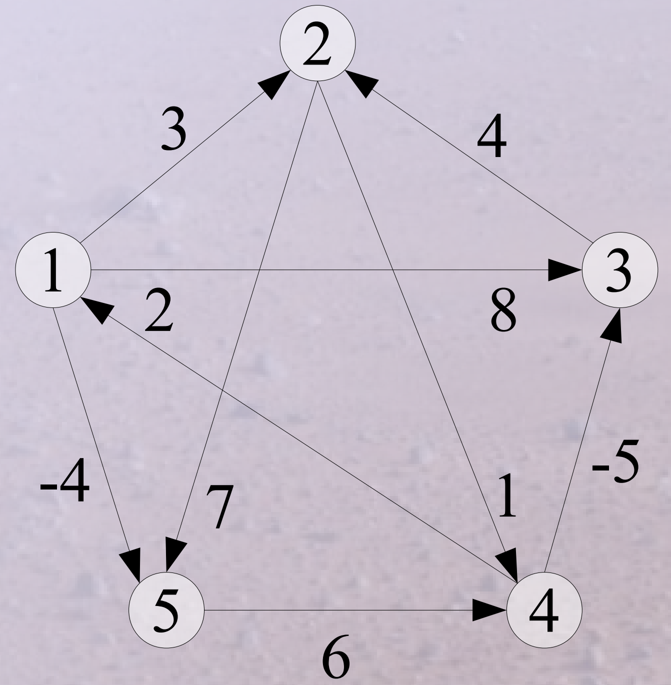
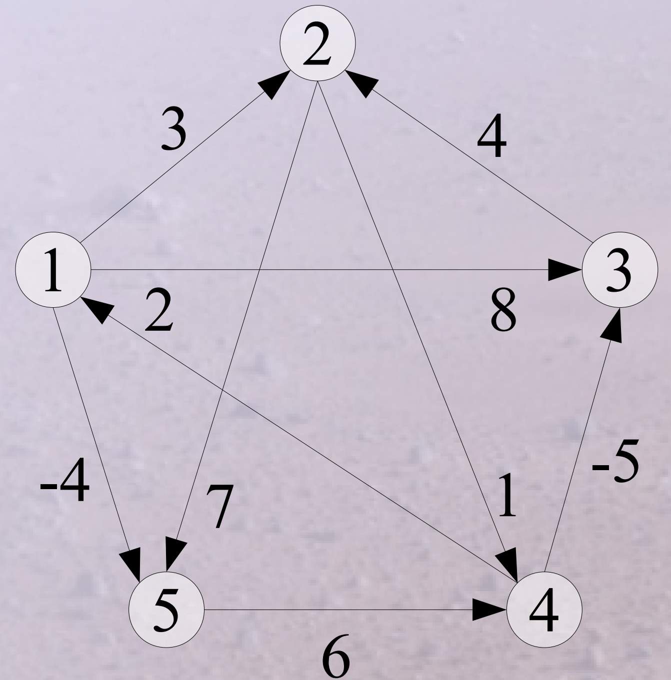
$$D^{(2)} = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$




```
Floyd-Warshall (W)
n = W.rows
D = W
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            d_ij = min (d_ij , d_ik + d_kj )
return D
```

$$D = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

Floyd-Warshall

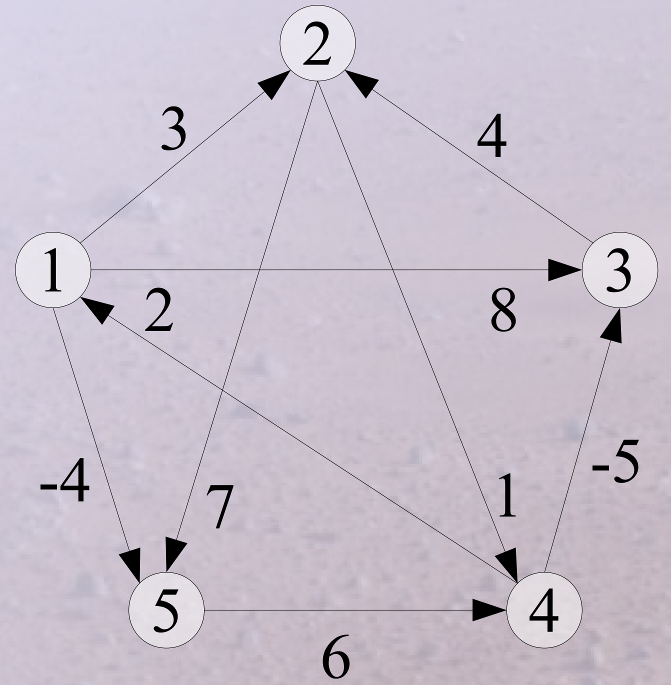


Floyd-Warshall (W)
$n = W.rows$
$D = W$
for $k = 1$ **to** n
  for $i = 1$ **to** n
    for $j = 1$ **to** n
      $d_{ij} = min\,(d_{ij}, d_{ik} + d_{kj})$
return D

$$D = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

$$D^{(3)} = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

Floyd-Warshall




```
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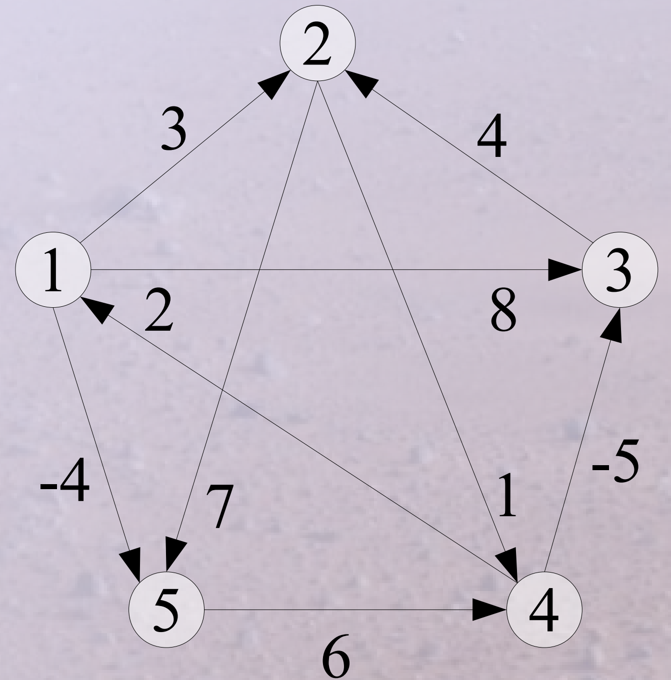
Floyd-Warshall

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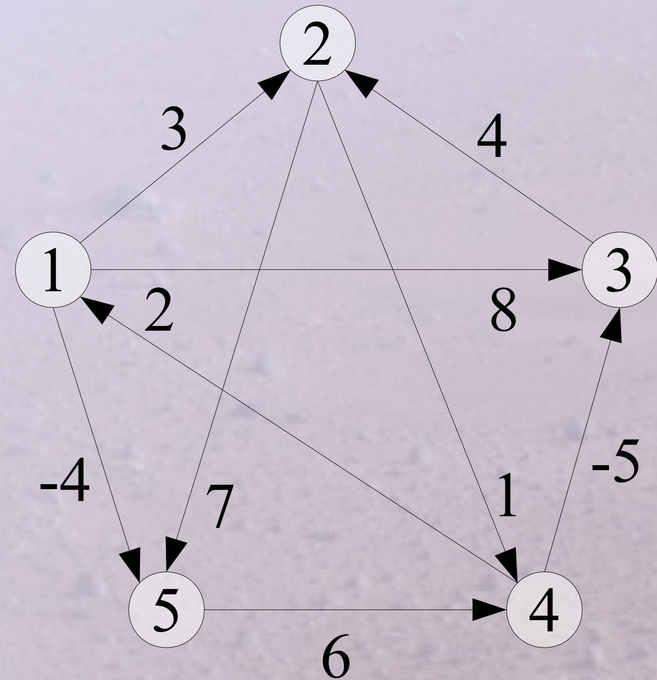
$$D^{(4)}=\begin{pmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

Floyd-Warshall



$$D = \begin{pmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

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Floyd-Warshall (W )
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for k = 1 to n
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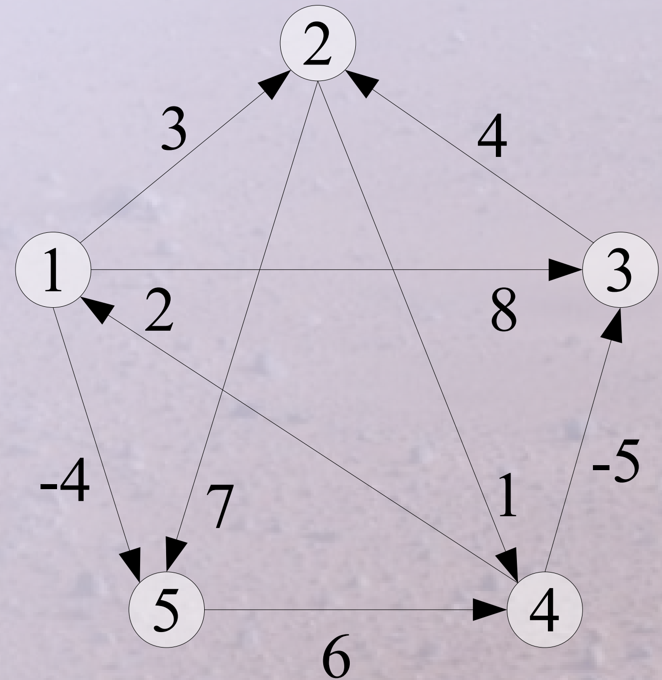


Floyd-Warshall (W)

$n = W.rows$

$D = W$

for $k = 1$ **to** n

  for $i = 1$ **to** n

    for $j = 1$ **to** n

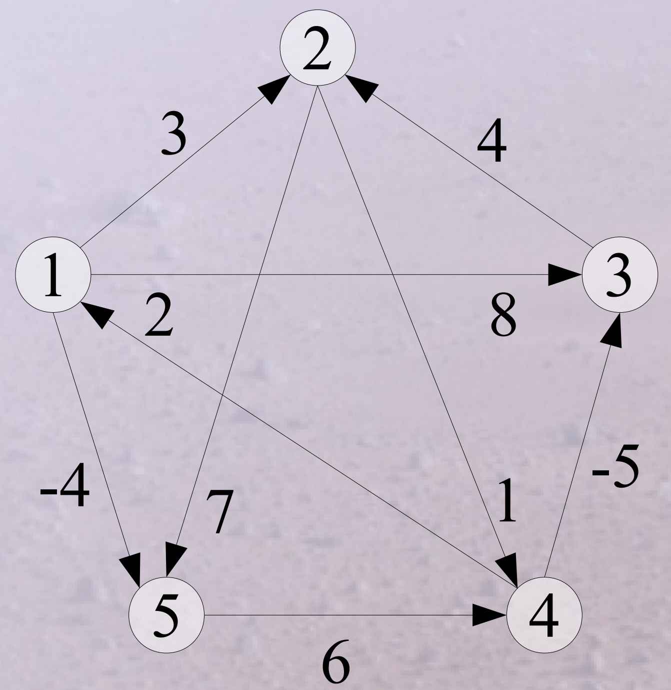
      $d_{ij} = min\,(d_{ij}\,,\, d_{ik} + d_{kj}\,)$

return D

$$D = \begin{pmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

$$D^{(5)} = \begin{pmatrix} 0 & 3 & -1 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

Floyd-Warshall




```
Floyd-Warshall (W)
n = W.rows
D = W
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            d_ij = min (d_ij , d_ik + d_kj )
return D
```

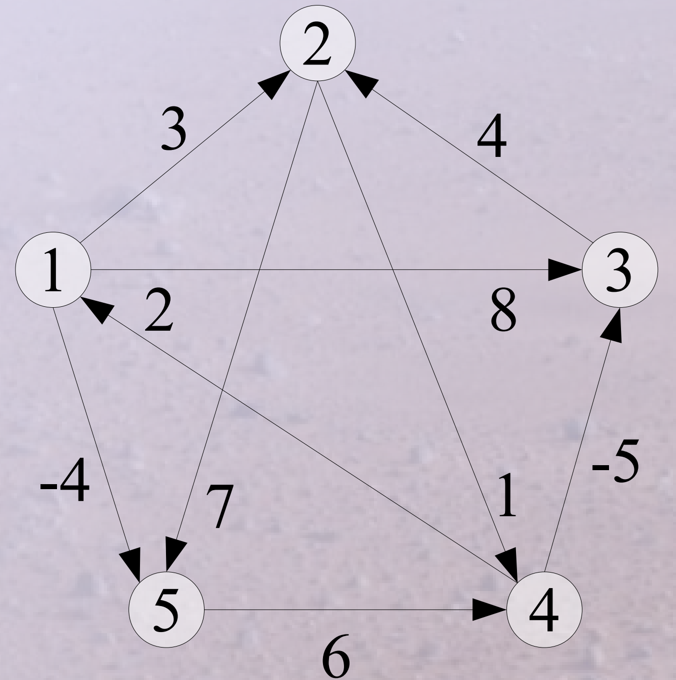
$$D = \begin{pmatrix} 0 & 3 & -1 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

Floyd-Warshall

- Kjøretid: $\Theta(n^3)$
  - Bedre enn Dijkstra fra hver node
  - Lave konstantledd
- Minne: $\Theta(n^2)$

```
Floyd-Warshall (W)
n = W.rows
D = W
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            d_ij = min (d_ij , d_ik + d_kj )
return D
```

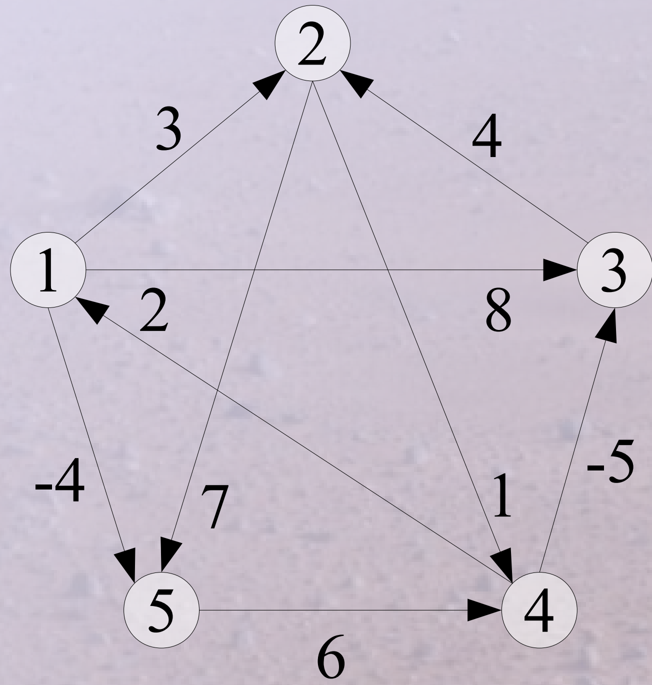
Floyd-Warshall




```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

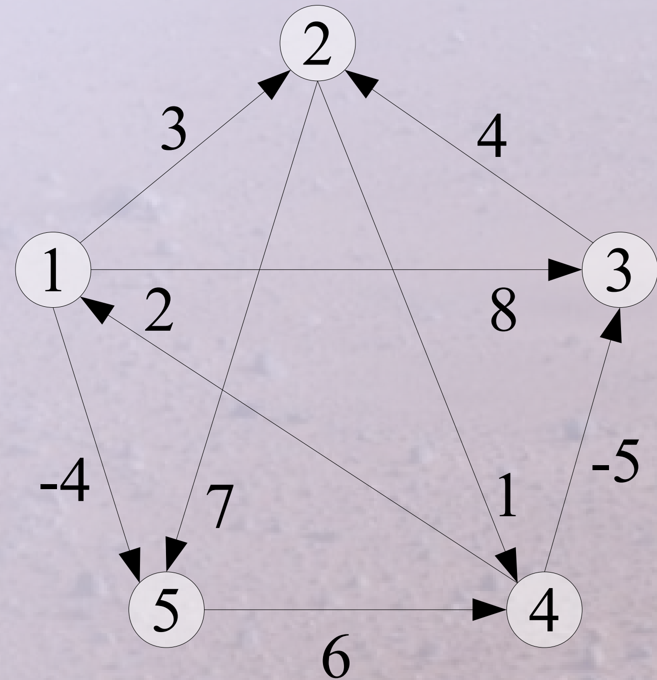
$$\Pi^{(0)} = \begin{pmatrix} \text{NIL} & 1 & 1 & \text{NIL} & 1 \\ \text{NIL} & \text{NIL} & \text{NIL} & 2 & 2 \\ \text{NIL} & 3 & \text{NIL} & \text{NIL} & \text{NIL} \\ 4 & \text{NIL} & 4 & \text{NIL} & \text{NIL} \\ \text{NIL} & \text{NIL} & \text{NIL} & 5 & \text{NIL} \end{pmatrix}$$

Floyd-Warshall




```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

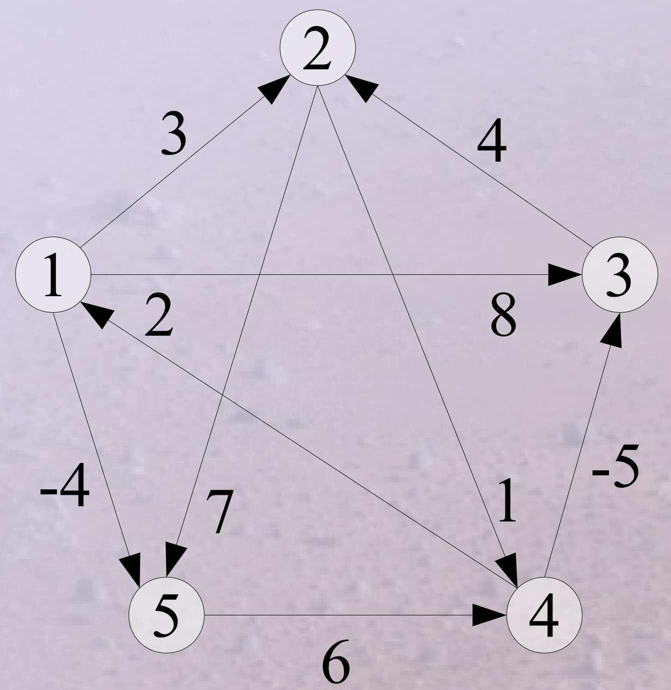
$$\Pi^{(0)} = \begin{pmatrix} NIL & 1 & 1 & NIL & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & NIL & NIL \\ 4 & NIL & 4 & NIL & NIL \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix} \quad \Pi^{(1)} = \begin{pmatrix} NIL & 1 & 1 & NIL & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & NIL & NIL \\ 4 & 1 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix}$$




```
Floyd-Warshall' (W, P)
  n = W.rows
  D = W
  Π = P
  for k = 1 to n
      for i = 1 to n
          for j = 1 to n
              if d_ik + d_kj < d_ij
                  d_ij = d_ik + d_kj
                  π_ij = π_kj
  return (D, Π)
```

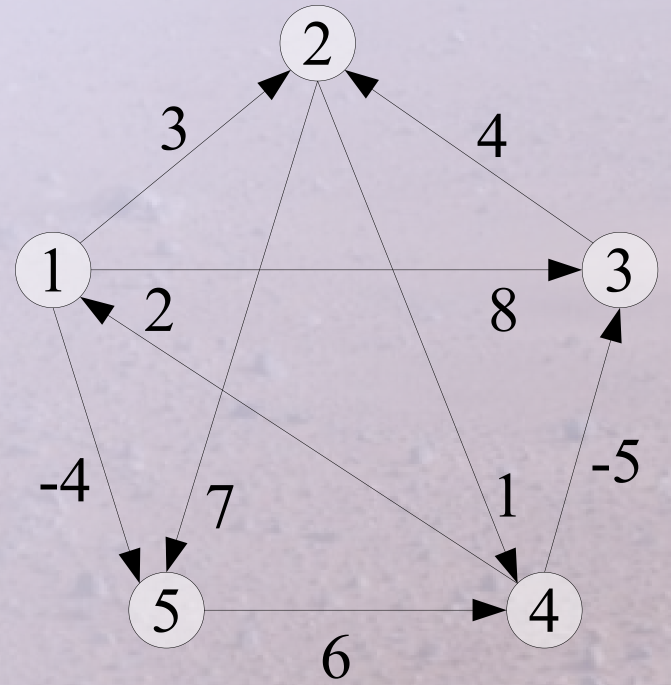
$$\Pi^{(1)} = \begin{pmatrix} NIL & 1 & 1 & NIL & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & NIL & NIL \\ 4 & 1 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix}$$

Floyd-Warshall



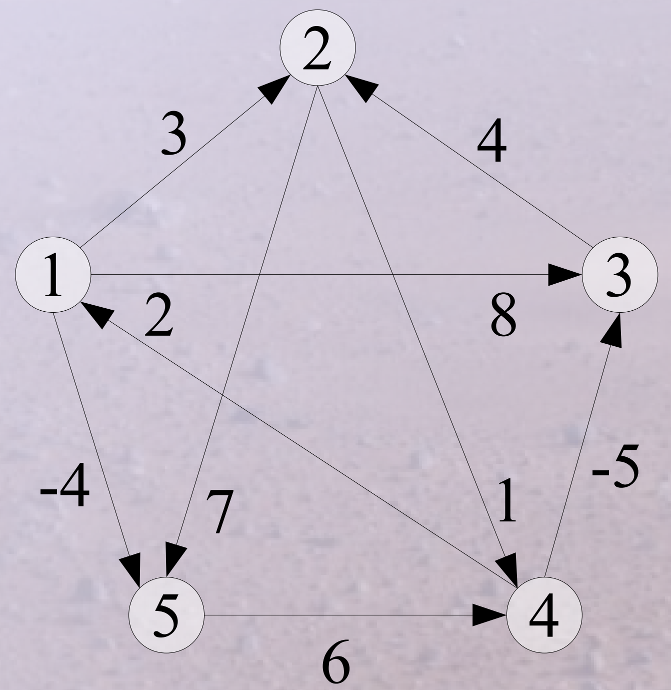

```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

$$\Pi^{(1)} = \begin{pmatrix} NIL & 1 & 1 & NIL & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & NIL & NIL \\ 4 & 1 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix} \quad \Pi^{(2)} = \begin{pmatrix} NIL & 1 & 1 & 2 & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & 2 & 2 \\ 4 & 1 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix}$$



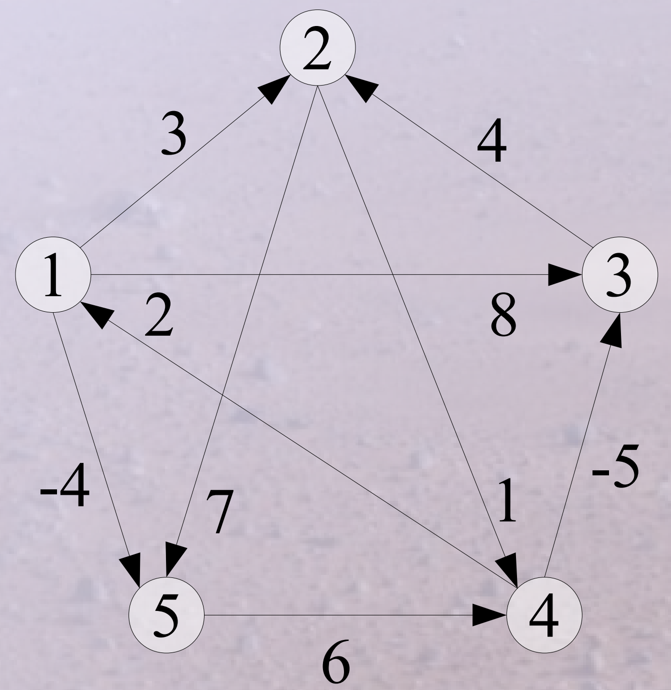

```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

$$\Pi^{(2)} = \begin{pmatrix} \text{NIL} & 1 & 1 & 2 & 1 \\ \text{NIL} & \text{NIL} & \text{NIL} & 2 & 2 \\ \text{NIL} & 3 & \text{NIL} & 2 & 2 \\ 4 & 1 & 4 & \text{NIL} & 1 \\ \text{NIL} & \text{NIL} & \text{NIL} & 5 & \text{NIL} \end{pmatrix}$$



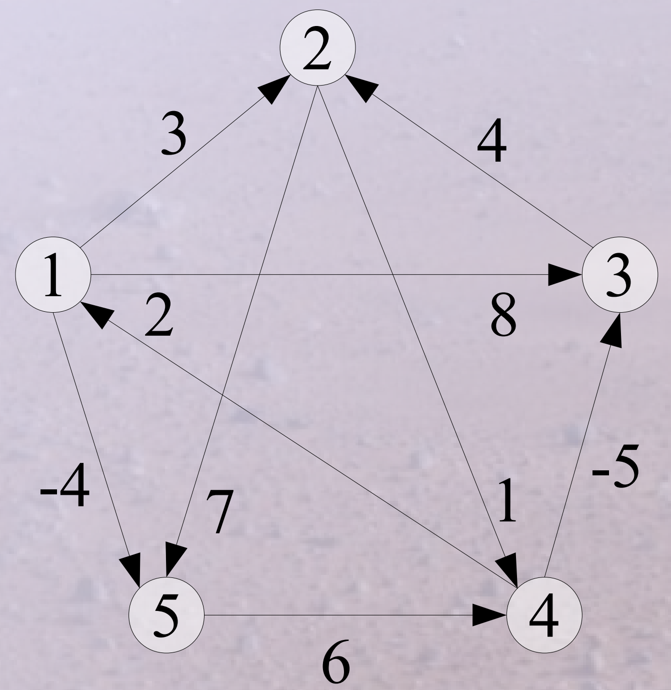

```
Floyd-Warshall' (W, P)
  n = W.rows
  D = W
  Π = P
  for k = 1 to n
      for i = 1 to n
          for j = 1 to n
              if d_ik + d_kj < d_ij
                  d_ij = d_ik + d_kj
                  π_ij = π_kj
  return (D, Π)
```

$$\Pi^{(2)} = \begin{pmatrix} NIL & 1 & 1 & 2 & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & 2 & 2 \\ 4 & 1 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix} \quad \Pi^{(3)} = \begin{pmatrix} NIL & 1 & 1 & 2 & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & 2 & 2 \\ 4 & 3 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix}$$



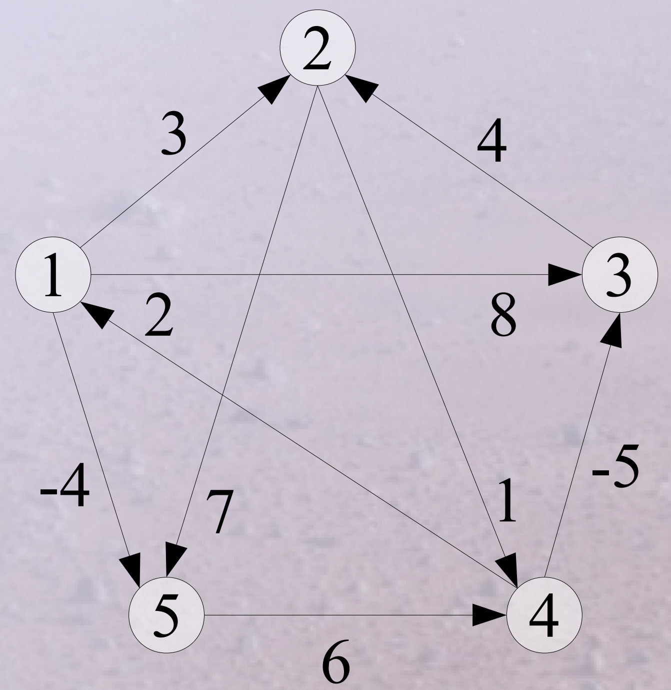

```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

$$\Pi^{(3)} = \begin{pmatrix} NIL & 1 & 1 & 2 & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & 2 & 2 \\ 4 & 3 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix}$$




```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

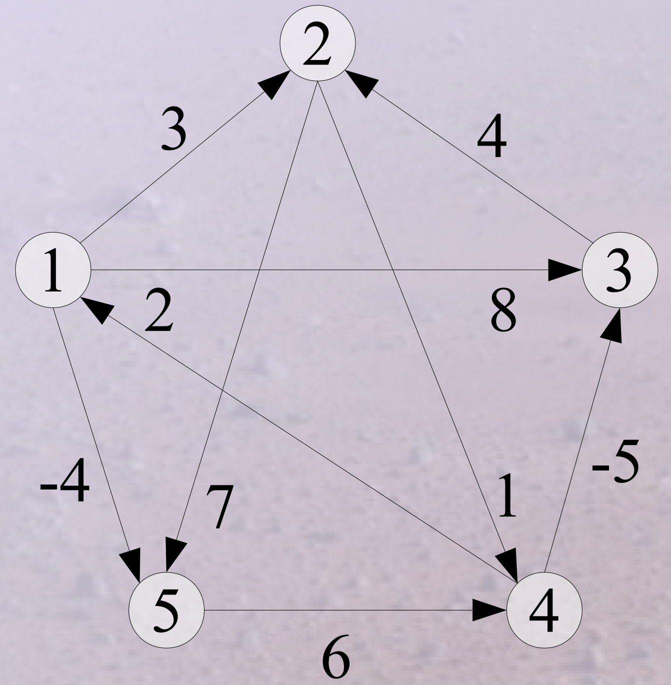
$$\Pi^{(3)}=\begin{pmatrix} NIL & 1 & 1 & 2 & 1 \\ NIL & NIL & NIL & 2 & 2 \\ NIL & 3 & NIL & 2 & 2 \\ 4 & 3 & 4 & NIL & 1 \\ NIL & NIL & NIL & 5 & NIL \end{pmatrix} \quad \Pi^{(4)}=\begin{pmatrix} NIL & 1 & 4 & 2 & 1 \\ 4 & NIL & 4 & 2 & 1 \\ 4 & 3 & NIL & 2 & 1 \\ 4 & 3 & 4 & NIL & 1 \\ 4 & 3 & 4 & 5 & NIL \end{pmatrix}$$




```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

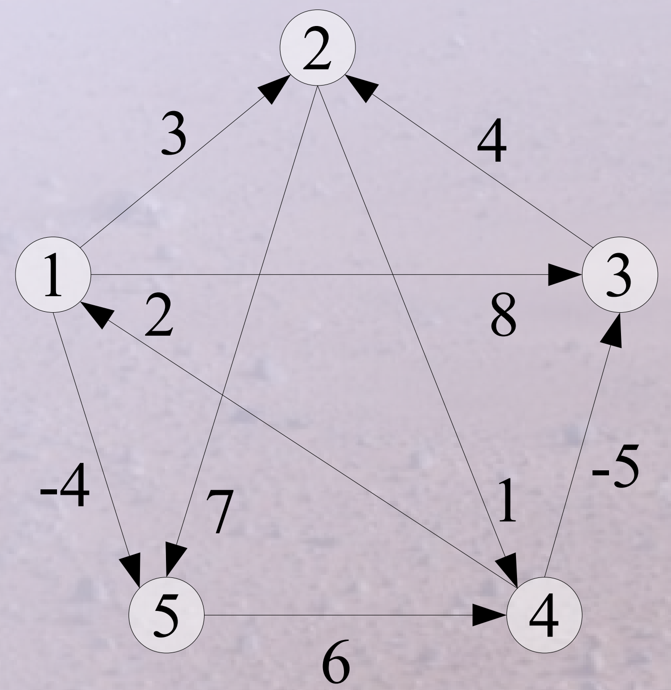
$$\Pi^{(4)} = \begin{pmatrix} \text{NIL} & 1 & 4 & 2 & 1 \\ 4 & \text{NIL} & 4 & 2 & 1 \\ 4 & 3 & \text{NIL} & 2 & 1 \\ 4 & 3 & 4 & \text{NIL} & 1 \\ 4 & 3 & 4 & 5 & \text{NIL} \end{pmatrix}$$

Floyd-Warshall




```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

$$\Pi^{(4)} = \begin{pmatrix} \text{NIL} & 1 & 4 & 2 & 1 \\ 4 & \text{NIL} & 4 & 2 & 1 \\ 4 & 3 & \text{NIL} & 2 & 1 \\ 4 & 3 & 4 & \text{NIL} & 1 \\ 4 & 3 & 4 & 5 & \text{NIL} \end{pmatrix} \quad \Pi^{(5)} = \begin{pmatrix} \text{NIL} & 3 & 4 & 5 & 1 \\ 4 & \text{NIL} & 4 & 2 & 1 \\ 4 & 3 & \text{NIL} & 2 & 1 \\ 4 & 3 & 4 & \text{NIL} & 1 \\ 4 & 3 & 4 & 5 & \text{NIL} \end{pmatrix}$$



$$\Pi^{(5)} = \begin{pmatrix} NIL & 3 & 4 & 5 & 1 \\ 4 & NIL & 4 & 2 & 1 \\ 4 & 3 & NIL & 2 & 1 \\ 4 & 3 & 4 & NIL & 1 \\ 4 & 3 & 4 & 5 & NIL \end{pmatrix}$$

```
Floyd-Warshall' (W, P)
n = W.rows
D = W
Π = P
for k = 1 to n
    for i = 1 to n
        for j = 1 to n
            if d_ik + d_kj < d_ij
                d_ij = d_ik + d_kj
                π_ij = π_kj
return (D, Π)
```

```
Print-All-Pairs-Shortest-Path (Π, i, j)
if i == j
    print i
elseif π_ij == NIL
    print "No path from " i "to " j " exists"
else Print-All-Pairs-Shortest-Path (Π, i, π_ij)
    print j
```

Oppsummering

- Korteste vei alle-til-alle, første forsøk:
  - Kjør en korteste vei én-til-alle algoritme fra hver node
  - Bellman-Ford
    - Veldig tregt
  - Dijkstra
    - Ikke negative kantvekter

- Korteste vei alle-til-alle, andre forsøk:
  - Sjekk om vi kan få en bedre vei ved å bruke en kant til

- Korteste vei alle-til-alle, tredje forsøk (Floyd-Warshall):
  - Finne beste vei ved å bruke de første k nodene
  - Sjekk for alle k om det er bedre å inkludere k på stien

Denne ukens øving

- Pipesortering