Interactive closest point

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A method for Registration of 3-D Shapes

- ICP is a ground stone of most registration algorithms to day
- Old from 1992
- General algorithm: point clouds, polygons to surfaces
- Local and Global Registration
Based on Euclidean distance

Distance to a triangle \( t \)

\[
d(q, r) = \sqrt{\sum_{i=1}^{n} (r_i - q_i)^2}
\]

\[
d(\mathbf{p}, t) = \min_{u+v+w=1} \|u\mathbf{r}_1 + v\mathbf{r}_2 + w\mathbf{r}_3 - \mathbf{p}\|
\]

The distance between point \( p \) and the triangle set \( T \)

\[
d(p, T) = \min_{i=1,...,N_t} d(p, t_i)
\]

The closest point \( y \) on triangle set \( T \) satisfies the equality

\[
d(p, y) = d(p, T)
\]
Working on parametric surfaces

- First is to split into simplex based representation then approximate with Euclidean distance
- Then approximate to the parametric curve/surface with Newton’s minimization
Pipeline of ICP

- Compute the closest points. For each point in \( N_p \) to shape \( X \)
- Compute registration. The translation vector and rotation vector for data \( P \)
- Apply translation and rotation
- Terminate iterations if mean-square error lower then the wanted threshold
Theorem: The iterative closest point algorithm always converges monotonically to a local minimum with respect to the mean-square distance objective function.
Q unit quaternion

\[ q_r = [q_0 q_1 q_2 q_3]^T \]

Sum = 0

\[ q_t = [q_4 q_5 q_6]^T \]

\[ Q = [q_r | q_t]^T \]

\[ \Delta \vec{q}_k = \vec{q}_k - \vec{q}_{k-1} \]

\[ \theta_k = \cos^{-1} \left( \frac{\Delta \vec{q}_k^t \Delta \vec{q}_{k-1}}{\|\Delta \vec{q}_k\| \|\Delta \vec{q}_{k-1}\|} \right) \]
An Accelerated ICP Algorithm
Global registration

- Can get stuck in a local minimum
- Uses fast convergence to pick the best starting values
- Some impossible cases
Experimental Results

- Good results
- All six degrees of freedom
- Relatively insensitive to minor local errors
- Gives good match even with the Gaussian noise up to 10% of model size