TEASER: Fast and Certifiable Point Cloud Registration

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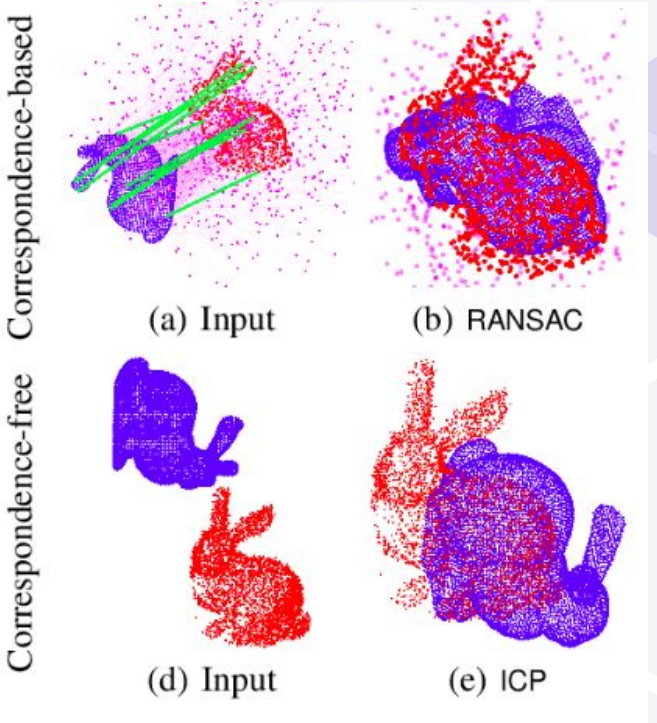
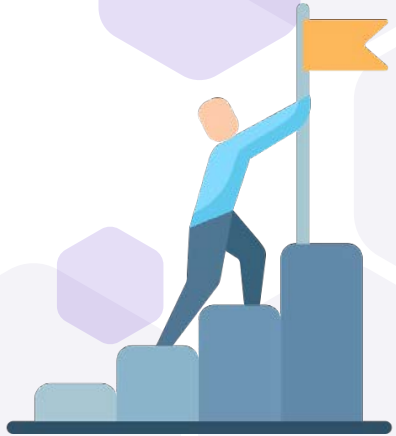
OVERVIEW

- Point cloud registration fundamental problem in computer vision and robotics
- Finding scale, rotation and translation
- Correspondences are either unknown or contains many outliers
- Commonly used algorithm relies either on initial guess or implicit assumption of outliers



WHAT WE WANT TO SOLVE?

- Solve registration globally without the need for an initial guess
- Tolerate a large number of outliers (e.g 99% outliers)
- Runs in polynomial time and fast in practice
- Provides formal performance guarantees



CONTRIBUTION

- General framework for decoupling scale, rotation and translation
- Show that the scale TLS estimation can be solved in polynomial time
- Theoretical results that certify the quality of the results by implemented algorithm TEASER
- Implement a faster version of TEASER named TEASER++
- An extensive evaluation in standard benchmarks and on real datasets

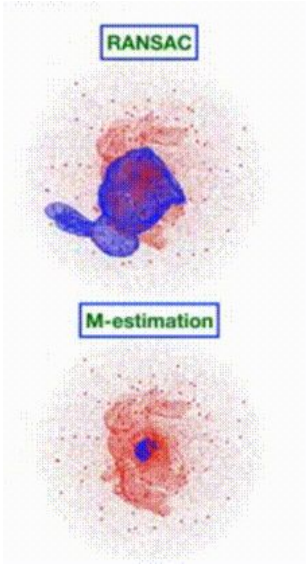
TWO METHODS

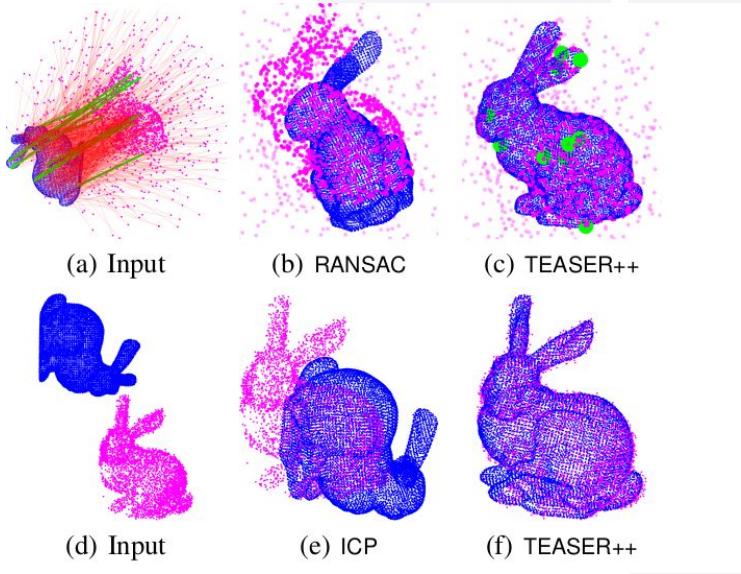
Correspondence-based

- Uses feature descriptor and estimator
- Computational complex and low guarantees

Simultaneous Pose and Correspondence

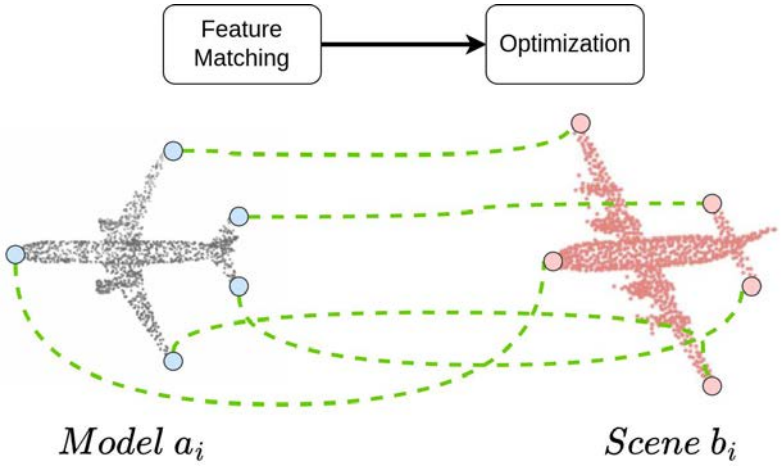
- Alternate between transformation and initial guess
- Computing transformation





SOLVES THE PROBLEM AS SPC, IN CONTRAST TO CORRESPONDENCE BASED PROBLEMS!

POINT CLOUD REGISTRATION PROBLEM



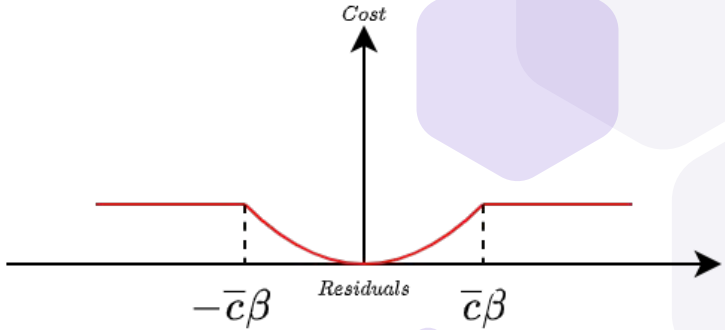
$$\min_{s>0, \boldsymbol{R}\in SO(3), \boldsymbol{t}\in\mathbb{R}^3} \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \|\boldsymbol{b}_i - s\boldsymbol{R}\boldsymbol{a}_i - \boldsymbol{t}\|^2$$

Scale *Rotation* *Translation*

95% of the correspondences can be incorrect!

Truncated Least Square Cost

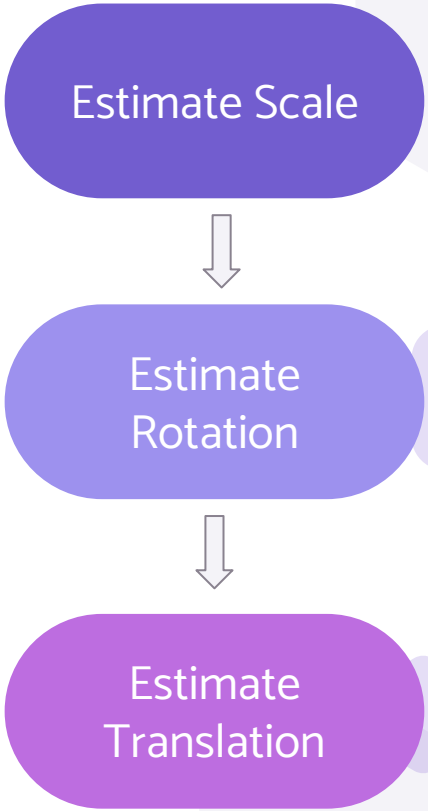
- Noise is unknown but bounded
- Outlier correspondence will not influence the optimization
- More challenging to solve globally



$$\min_{s>0,\boldsymbol{R}\in\mathrm{SO}(3),\boldsymbol{t}\in\mathbb{R}^3} \sum_{i=1}^{N} \min\left(\frac{1}{\beta_i^2}\left\|\boldsymbol{b}_i - s\boldsymbol{R}\boldsymbol{a}_i - \boldsymbol{t}\right\|^2,\ \bar{c}^2\right), \quad (6)$$

TEASER

- Estimate the scale
- Use scale and to to estimate the rotation
- Use both scale and rotation to estimate the translation



INVARIANT MEASUREMENTS

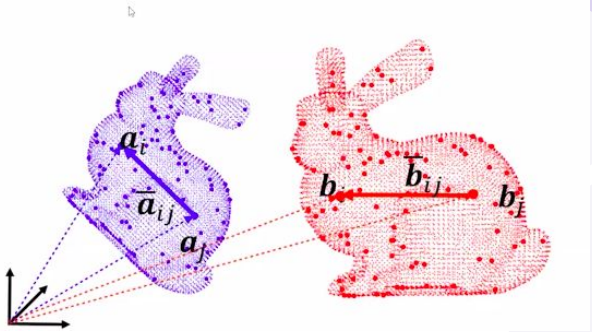
- Proposing a new approach to decouple estimation of scale, translation and rotation
- Reformulate the measurements to obtain quantities that are invariant to a subset of transformation (using TIM and TRIM)

TRANSLATION AND ROTATION INVARIANT MEASUREMENTS (TRIM)

- **Insight**: norm of relative positions are translation and rotation invariant

$$\|\bar{\boldsymbol{b}}_{ij}\| = \|s\boldsymbol{R}\bar{\boldsymbol{a}}_{ij} + \boldsymbol{o}_{ij} + \boldsymbol{\epsilon}_{ij}\|.$$

$$\|\bar{\boldsymbol{b}}_{ij}\| = \|s\boldsymbol{R}\bar{\boldsymbol{a}}_{ij}\| + \tilde{o}_{ij} + \tilde{\epsilon}_{ij},$$



- Dividing by $\|\bar{\boldsymbol{a}}_{ij}\|$ we can get an expression dependent on s: $s_{ij} = s + o_{ij}^{s} + \epsilon_{ij}^{s},$

TRIM FOR SOLVING SCALE

$$\hat{s} = \arg\min_{s} \sum_{k=1}^{K} \min\left(\frac{(s - s_k)^2}{\alpha_k^2}, \bar{c}^2 \right),$$

- Adaptive voting for achieving polynomial time
- Solution is trivial
  - enumerate
  - sort
  - vote

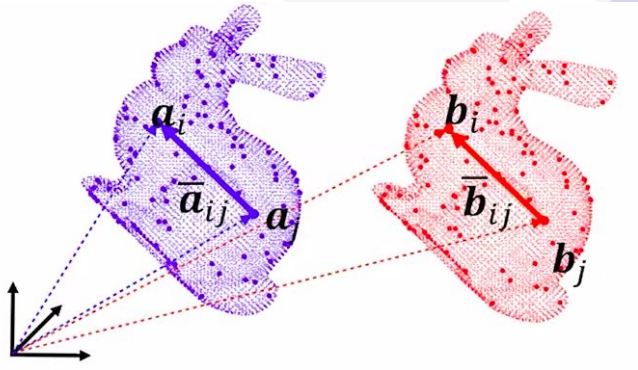
Translation Invariant Measurements (TIM)

- Insight: Absolute positions depend on the translation, the **relative positions** are invariant to translation

$$\bar{\boldsymbol{b}}_{ij} = s\boldsymbol{R}\bar{\boldsymbol{a}}_{ij} + \boldsymbol{o}_{ij} + \boldsymbol{\epsilon}_{ij}, \qquad \text{(TIM)}$$

$$\boldsymbol{b}_j - \boldsymbol{b}_i = s\boldsymbol{R}(\boldsymbol{a}_j - \boldsymbol{a}_i) + (\boldsymbol{o}_j - \boldsymbol{o}_i) + (\boldsymbol{\epsilon}_j - \boldsymbol{\epsilon}_i),$$

- Only depends to solve two unknowns rotation and scale



TIM FOR SOLVING ROTATION

$$\hat{\boldsymbol{R}} = \underset{\boldsymbol{R}\in \mathrm{SO}(3)}{\arg\min} \sum_{k=1}^{K} \min\left(\frac{\|\bar{\boldsymbol{b}}_k - \hat{s}\boldsymbol{R}\bar{\boldsymbol{a}}_k\|^2}{\delta_k^2} \,,\, \bar{c}^2 \right),$$

- Rotation Estimation as **QCQP**:

$$\min_{\boldsymbol{q}\in\mathbb{S}^3} \sum_{k=1}^{K} \min\left(\frac{\|\hat{\boldsymbol{b}}_k - \boldsymbol{q}\circ\hat{\boldsymbol{a}}_k\circ\boldsymbol{q}^{-1}\|^2}{\delta_k^2} \,,\, \bar{c}^2 \right),$$

- Replacing the SO(3) to simpler set 4D sphere
- Still a non-convex problem!

TIM FOR TRANSLATION

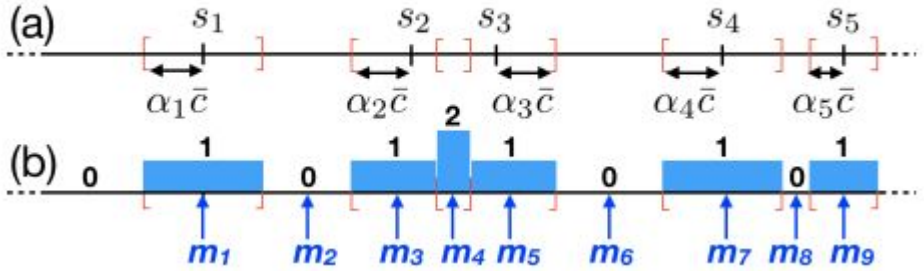
$$\hat{t}_j = \arg\min_{t_j} \sum_{i=1}^{N} \min\left(\frac{(t_j - [\boldsymbol{b}_i - \hat{s}\hat{\boldsymbol{R}}\boldsymbol{a}_i]_j)^2}{\beta_i^2}, \bar{c}^2 \right),$$

- Uses also adaptive voting for finding optimal translation
- Computing three translations for x, y and z

Scale and Translation: Adaptive Voting

Algorithm 2: Adaptive Voting.

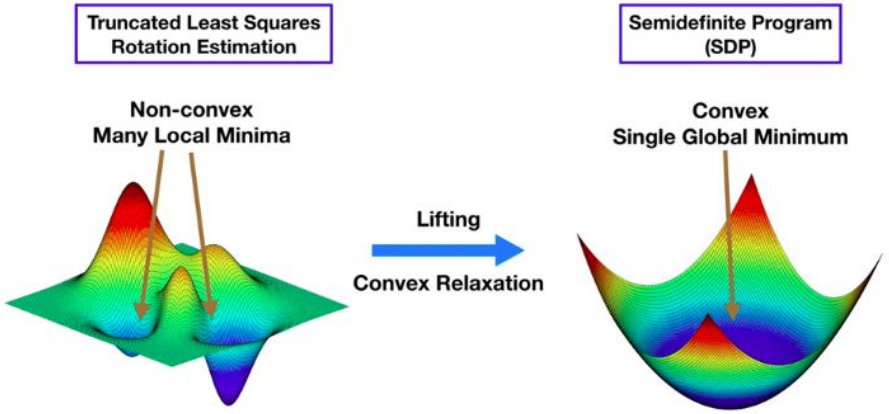
Input: $s_k, \alpha_k, \bar{c}$;
Output: $\hat{s}$, scale estimate solving (11);
% Define boundaries and sort
$\boldsymbol{v} = \text{sort}([s_1 - \alpha_1\bar{c}, s_1 + \alpha_1\bar{c}, \ldots, s_K - \alpha_K\bar{c}, s_K + \alpha_K\bar{c}])$
% Compute middle points
$m_i = \frac{\boldsymbol{v}_i + \boldsymbol{v}_{i+1}}{2}$ for $i = 1, \ldots, 2K-1$
% Voting
for $i = 1, \ldots, 2K-1$ **do**
  $\mathcal{I}_i = \emptyset$
  for $k = 1, \ldots, K$ **do**
    if $m_i \in [s_k - \alpha_k\bar{c}, s_k + \alpha_k\bar{c}]$ **then**
      $\mathcal{I}_i = \mathcal{I}_i \cup \{k\}$ % add to consensus set
    end
  end
end
% Enumerate consensus sets and return best
return: $\hat{s}$ from Eq. (14).



Rotation: Semidefinite Relaxation Problem

- Problem of solving rotation known as robust rotation search problem
- Using tight SDP
- Large scale SDP are computationally expensive, worst case do not being solved in polynomial time

WHAT NOW?



TEASER++

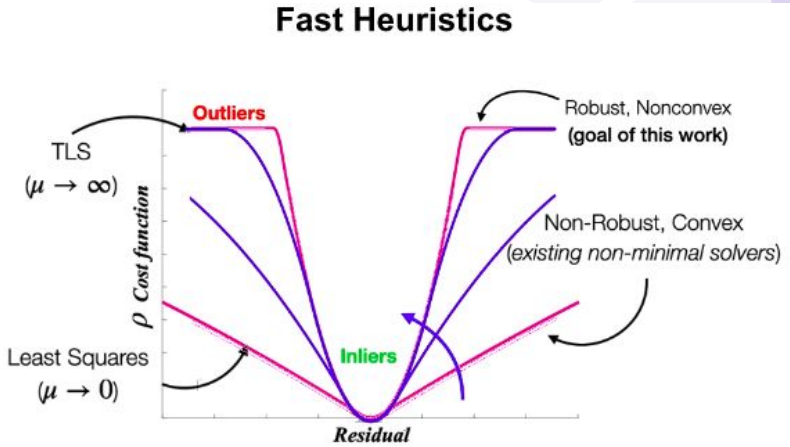
Estimate Scale → Adaptive Voting

Estimate Rotation →  GNC-TLS

Estimate Translation → Adaptive Voting

GNC-TLS: TEASER++

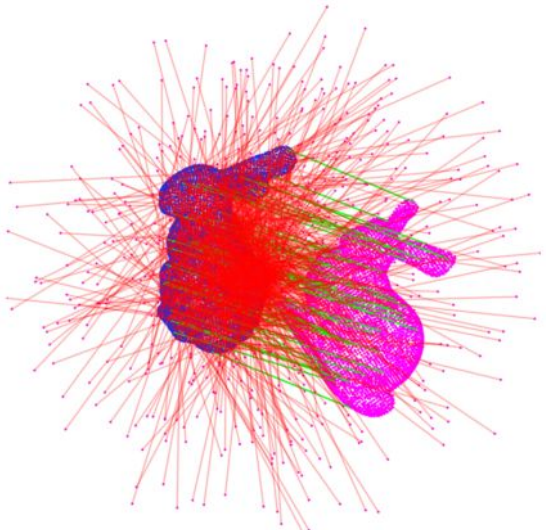
- Iterative method that uses TLS
- Three times faster than TEASER
- Removes large amount of outliers

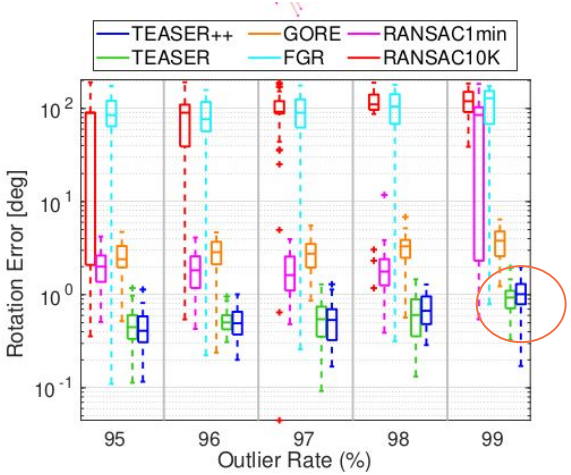


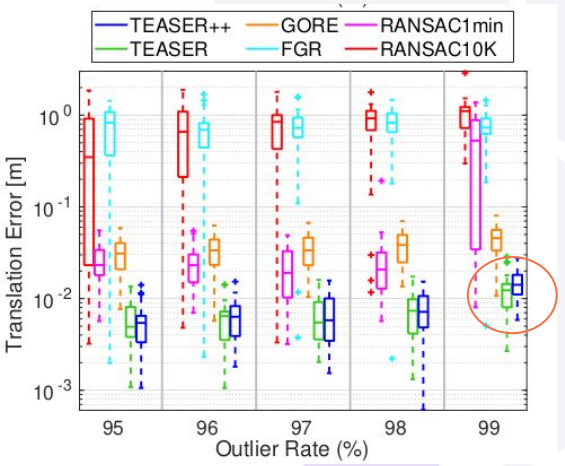
Semidefinite Relaxation Problem

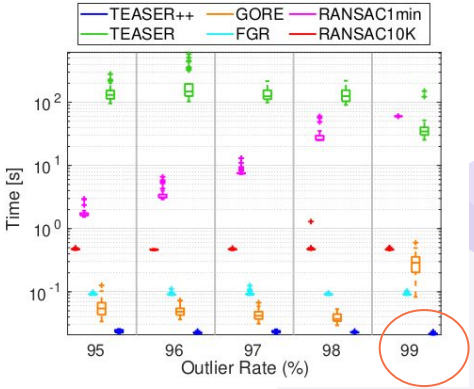
- Problem of solving rotation known as robust rotation search problem
- Solved in polynomial time
- Large scale SDP are computationally expensive

Benchmark



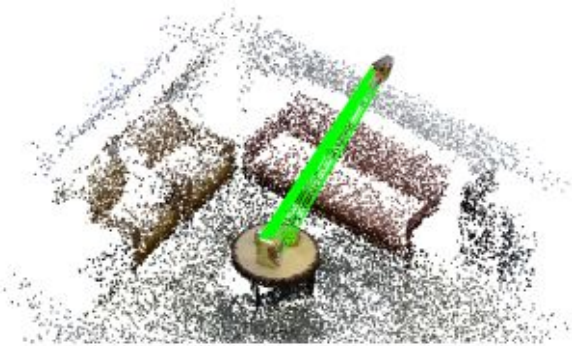

TEASER++
TEASER
GORE
FGR
RANSAC1min
RANSAC10K
Rotation Error [deg]
10^{2}
10^{1}
10^{0}
10^{-1}
95
96
97
98
99
Outlier Rate (%)


TEASER++
TEASER
GORE
FGR
RANSAC1min
RANSAC10K
Translation Error [m]
10^{0}
10^{-1}
10^{-2}
10^{-3}
95
96
97
98
99
Outlier Rate (%)


TEASER++
TEASER
GORE
FGR
RANSAC1min
RANSAC10K
Time [s]
10^{2}
10^{1}
10^{0}
10^{-1}
95
96
97
98
99
Outlier Rate (%)

Application 1: Object Pose Estimation

FPHP

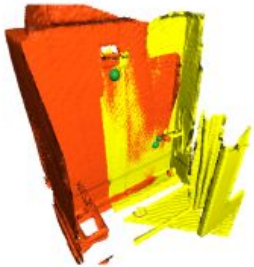


TEASER

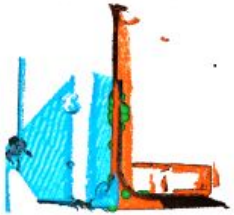
LOCALIZATION

Application 2: RGB-D Scan Matching

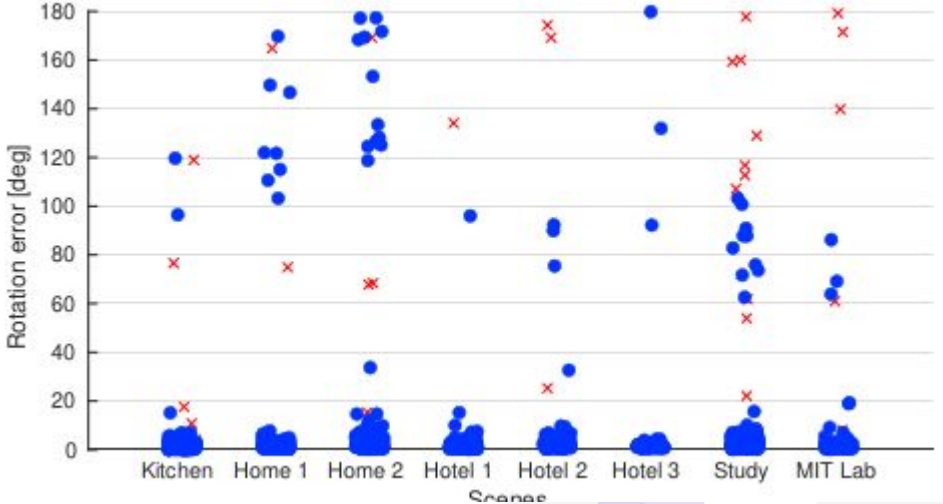
(a) Ground truth (top-down view)



(b) Ground truth (side view)



(c) TEASER++'s estimate (top-down view)



THANKS!

Any questions?